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# • FRACTALS – WHAT THEY CAN TELL YOU ABOUT YOUR HYDROCARBON VOLUME – NORTH SEA CASE STUDIES

• Steve Cuddy



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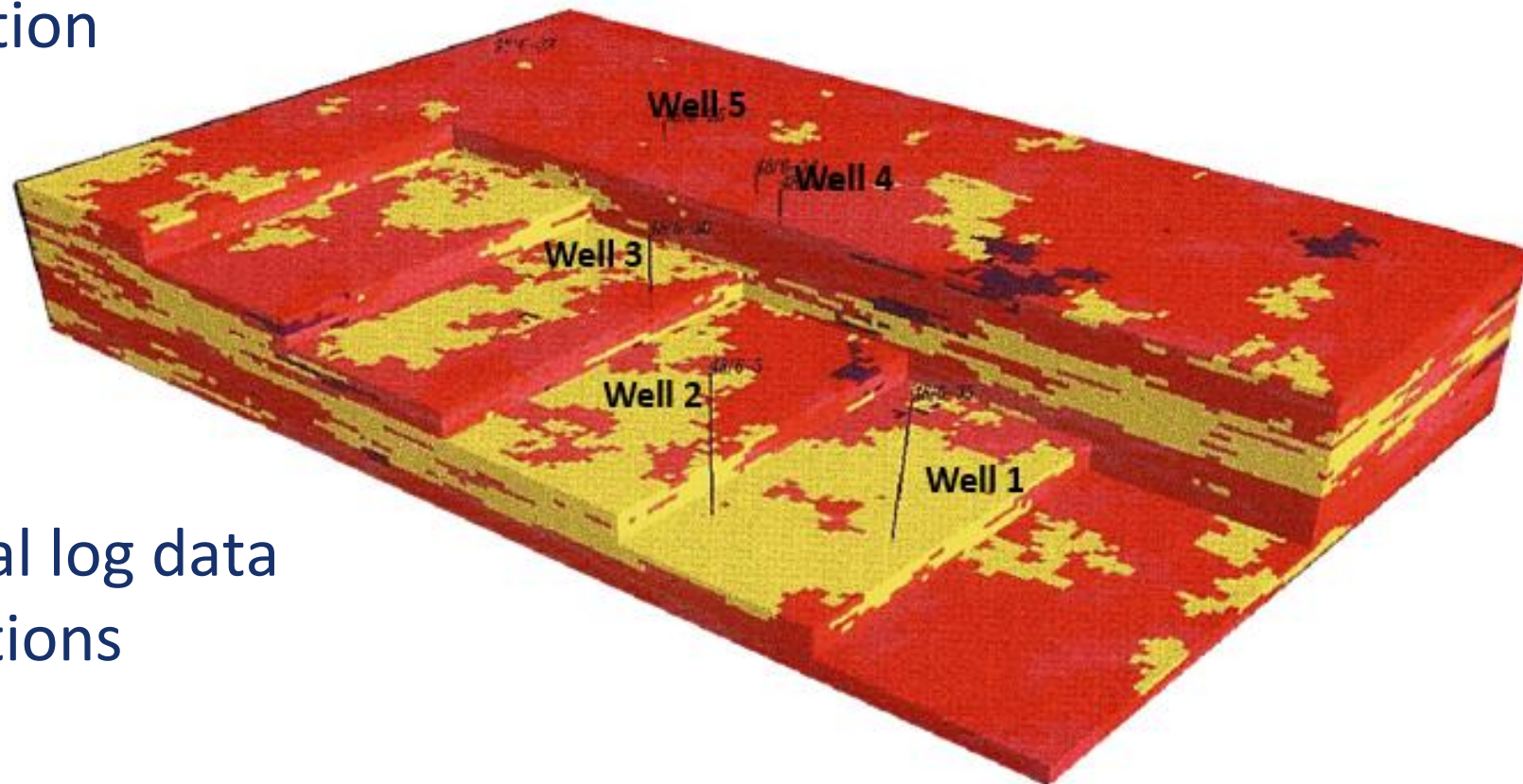
# Outline

- How we determine a reservoir's hydrocarbon volume
- How fractals make this easy
- Demonstrated using several North Sea case studies

# Why we need a reservoir model

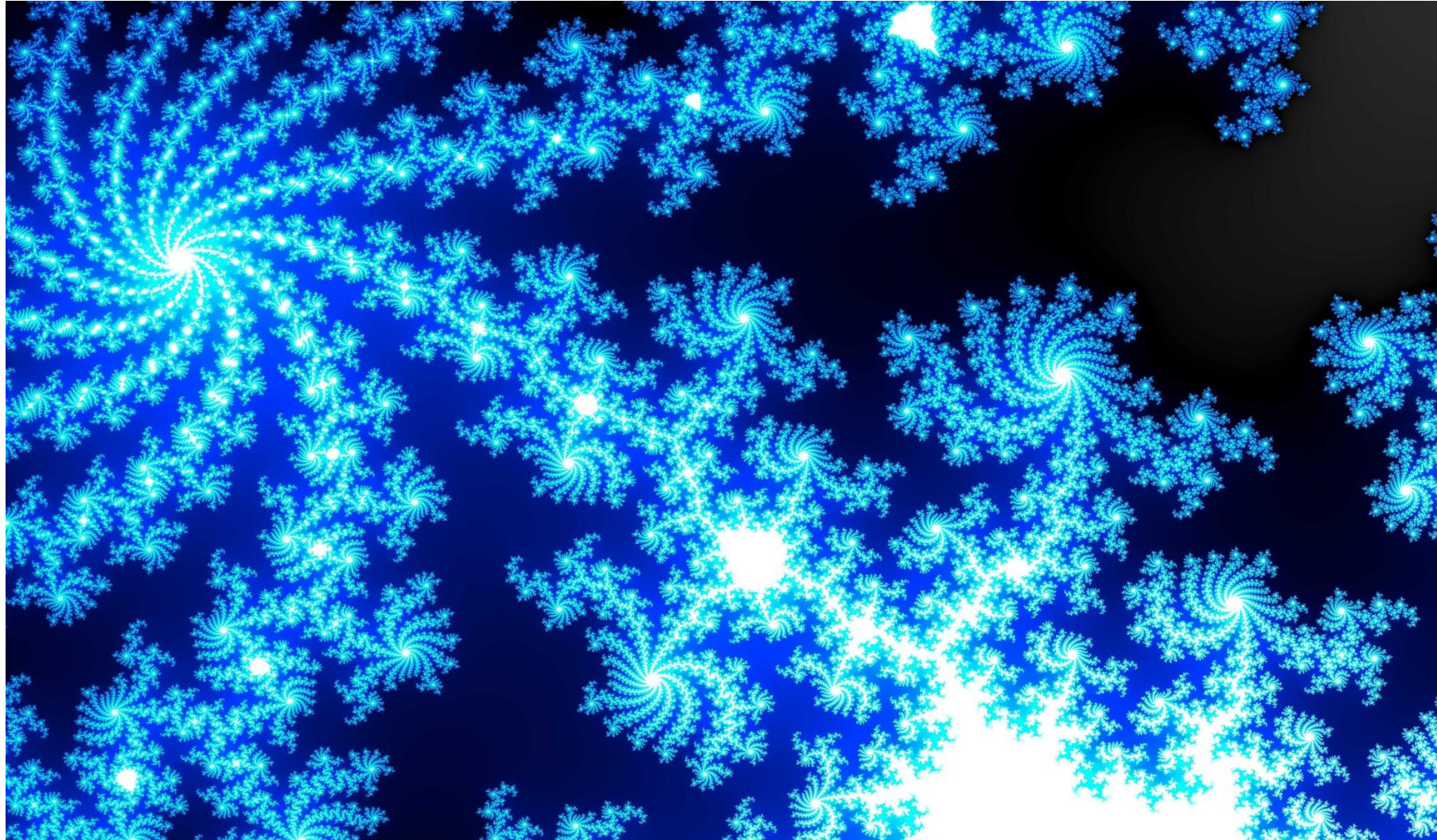
The 3D reservoir model is required to calculate hydrocarbon in place and for dynamic modelling

The model requires fluid contacts, net reservoir cut-off and a water saturation vs. height function



Limited core and electrical log data available at the well locations

# Fractals and Reservoir Description



# Fractals on the Small Scale



Snowflakes

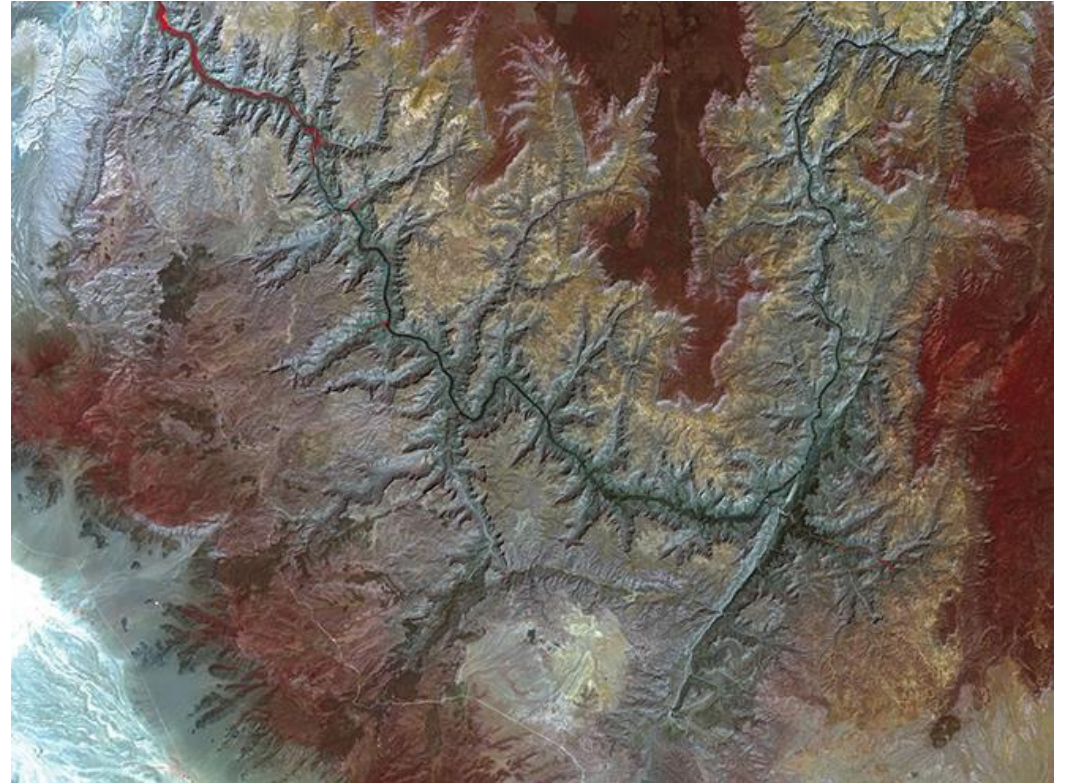


Roman Cauliflower

# Fractals on the Big Scale



Himalayas



Canyons

# Fractals on the Really Big Scale

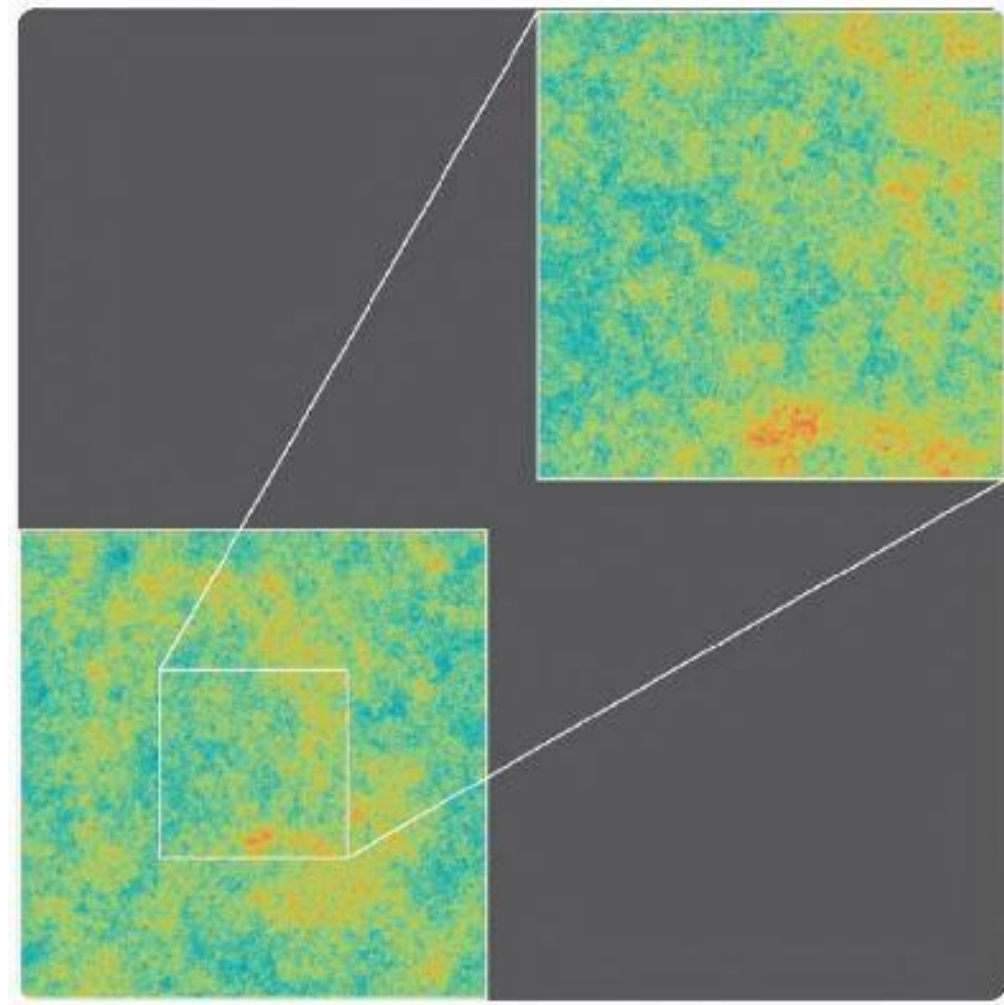
The cosmic microwave background is scale invariant

If we zoom in the patterns are indistinguishable

These patterns give rise to galactic superclusters

Galactic superclusters are built up from galaxies

The universe is fractal



Prof. Brian Cox – 'Forces of Nature' 2016



# What are Fractals?

A fractal is a never-ending pattern

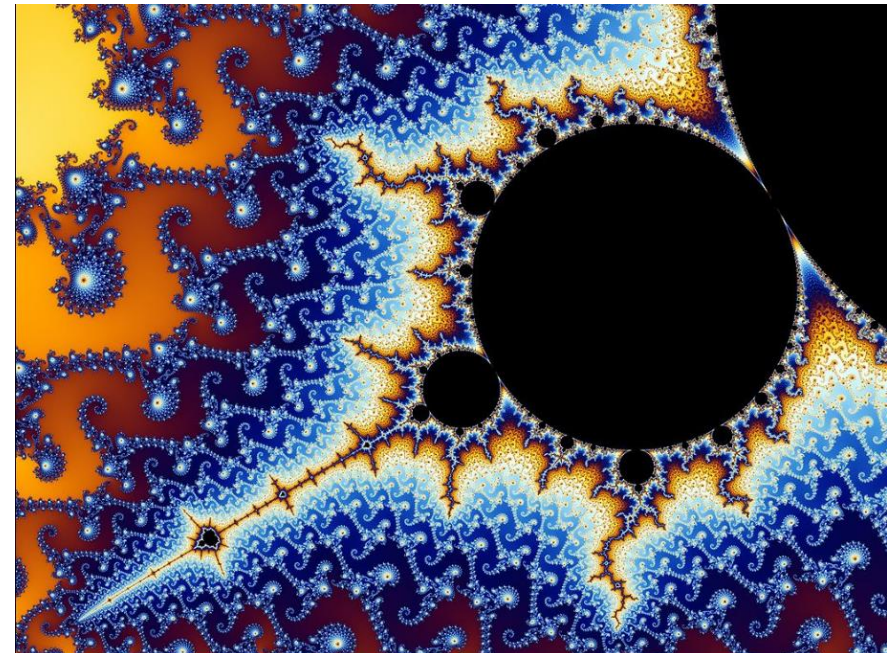
Fractals are infinitely complex patterns that look the same at **every** scale

They are created by **simple repeating** process

Benoit B. Mandelbrot set 

Other names for fractals are

- Self-similarity
- Scale invariance



# Why Fractals are Useful

Fractals are objects where their parts are similar to the whole except for scale

A simple repeating process can create a complex object

Many complex objects can be described by fractals

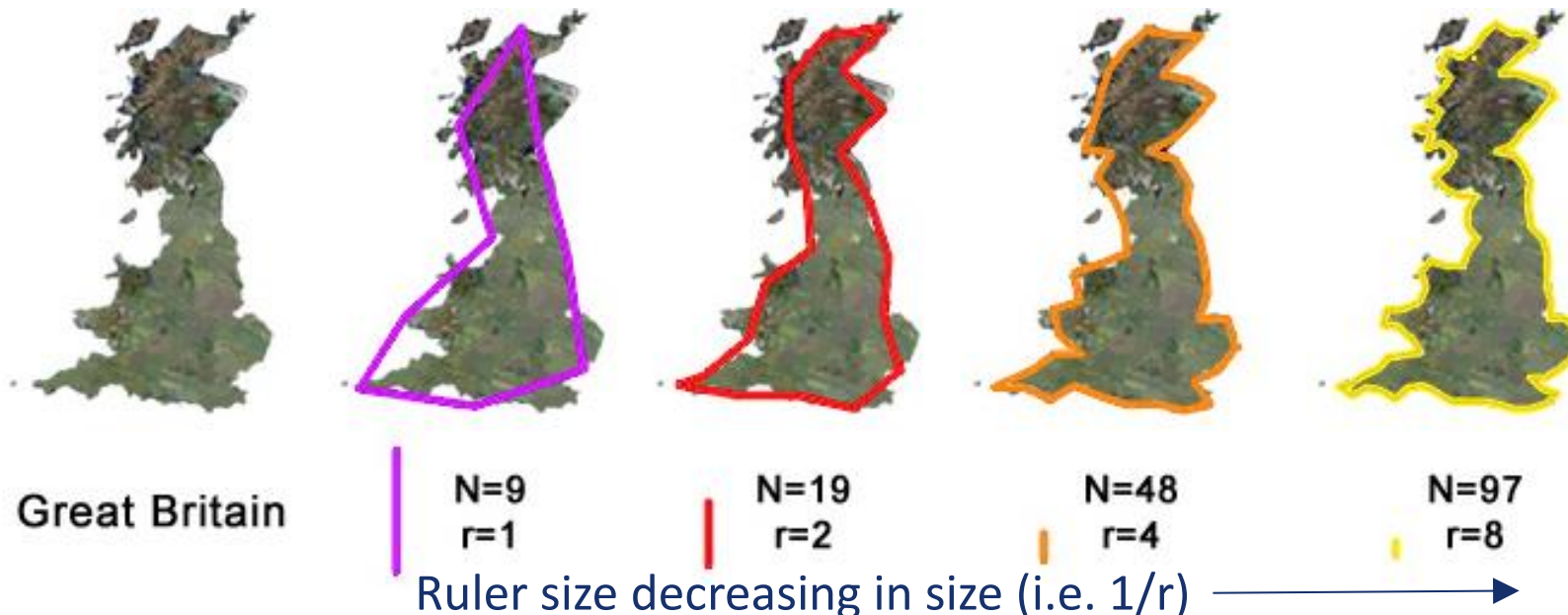
Mathematically simple



# How to verify if something is a fractal

Coastlines show more detail, the closer you zoom in

The length of Great Britain's coastline ( $N$ ) depends on the **length** of your ruler ( $r$ )

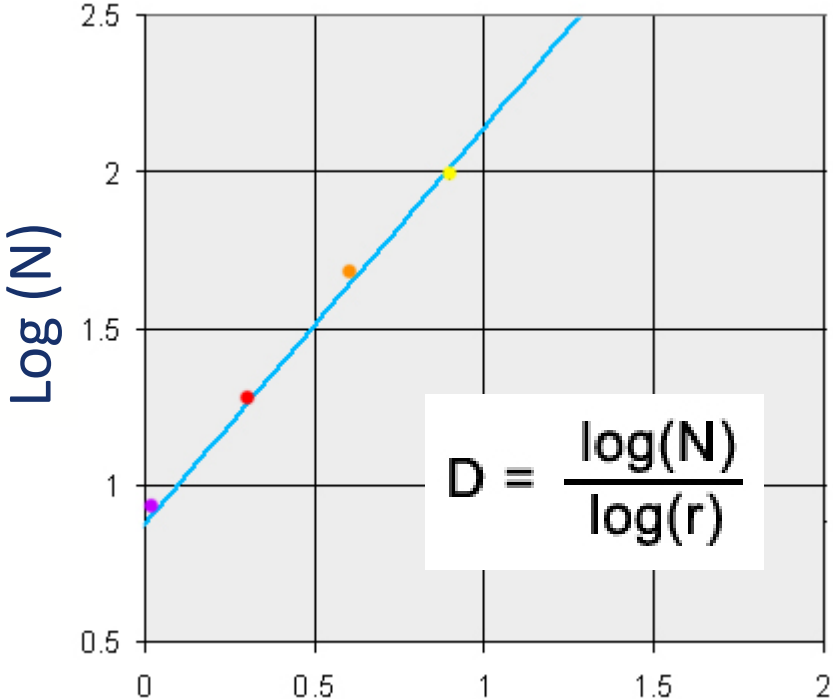


# Coastline Fractals

As the ruler shrinks the measured coastline increases

If the coastline is fractal the relationship between  $r$  and  $N$  is **linear** when plotted using log scales

$D$  = fractal dimension



Great Britain



$N=9$   
 $r=1$



$N=19$   
 $r=2$



$N=48$   
 $r=4$



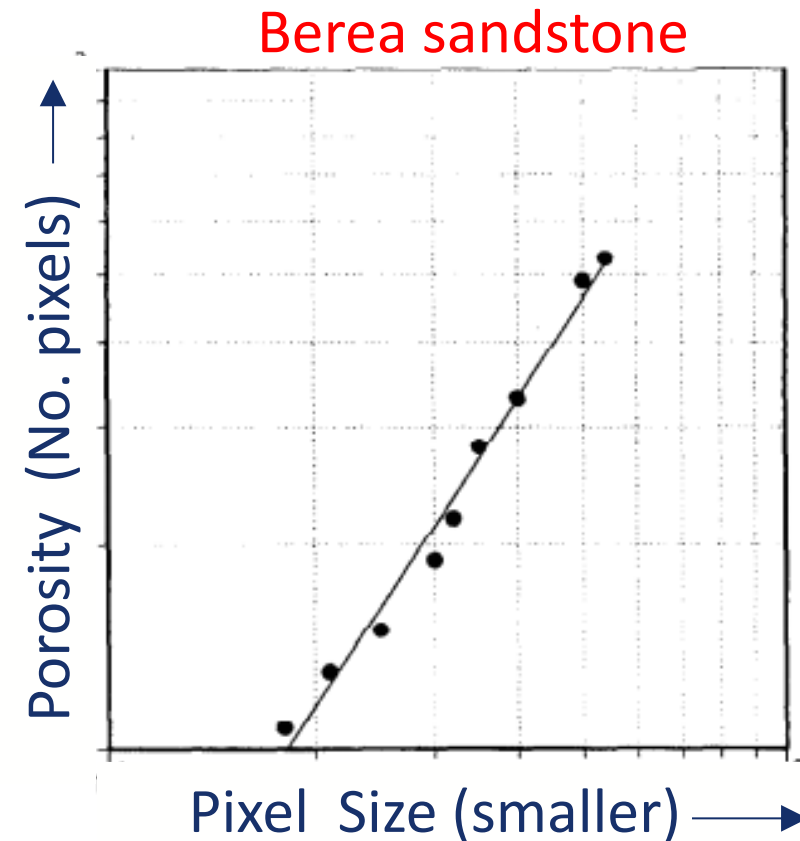
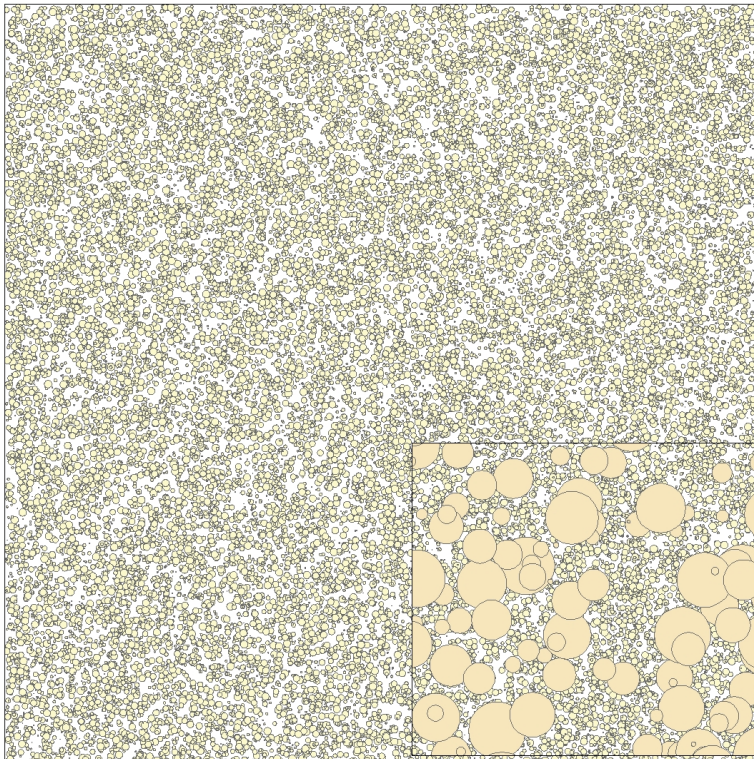
$N=97$   
 $r=8$

Ruler size decreasing in size (i.e.  $1/r$ )  $\longrightarrow$

# Fractals in reservoir rocks

Thin sections of reservoir rocks are imaged with a scanning electron microscope (SEM )

For different magnifications the number of pixels representing porosity are counted



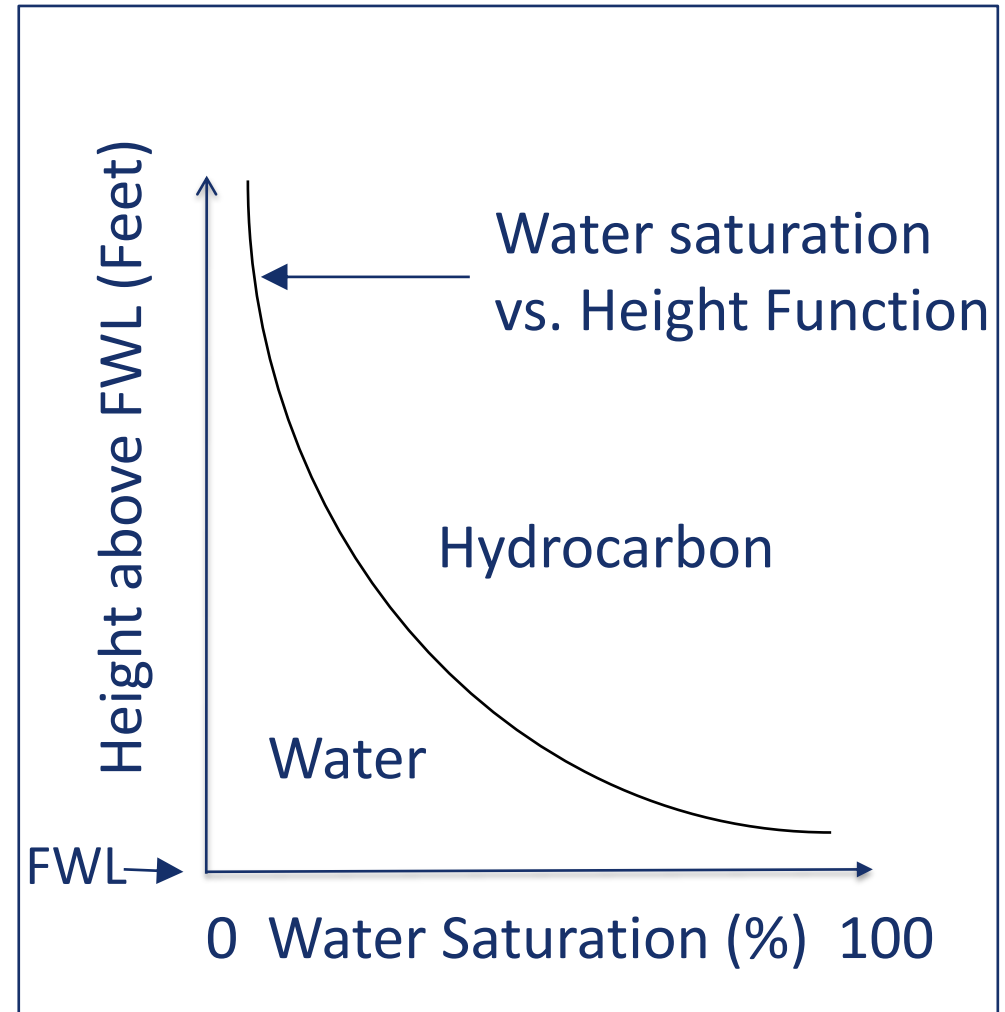
# The reservoir model needs a $S_w$ vs. Height Function

Used to initialize the 3D reservoir model

Tells us how water saturation varies as a function of the height above the Free Water Level (FWL)

Tells us how the formation porosity is split between hydrocarbon and water

Tells us the shape of the transition zone



# What a Good Saturation Height Function Requires

Three independent sources of fluid distribution data are consistent

- Formation pressure data
- Electrical log data
- Core data

Must account for varying permeability and fluid contacts

Must upscale correctly

Should be easy to apply

# Fractals describe the rock pore network

The rock pore space can be described by the fractal formula

$$V = r^{(3-Df)}$$

Where:

$V$  Pore space in rock volume

$r$  Radius of the rock capillaries

$Df$  Fractal dimension (non-integer constant)

This reduces to

$$BVW = aH^b$$

Where:

$BVW$  Volume of capillary bound water in the rock

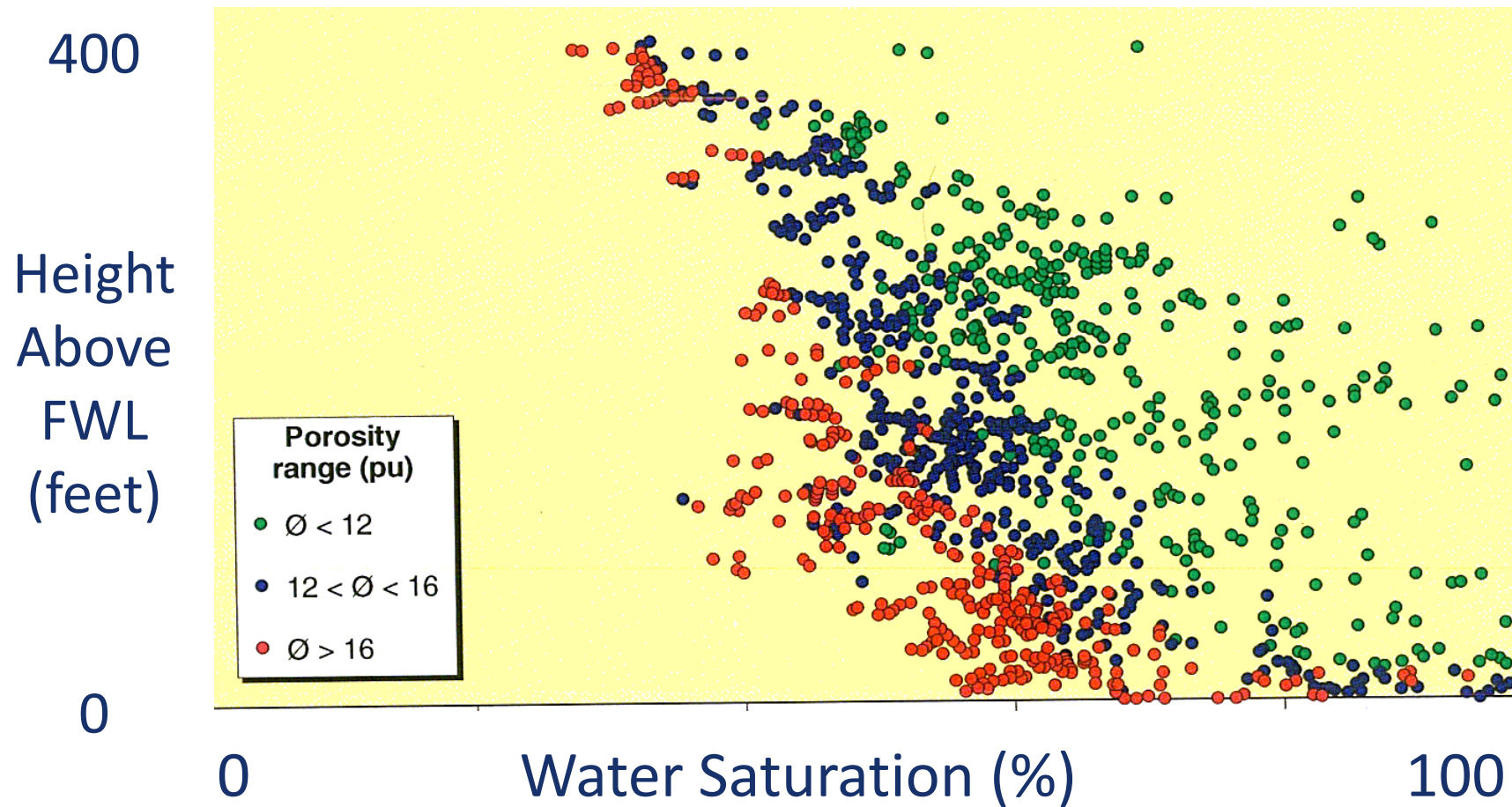
$H$  Height above the free water level

$a$  &  $b$  constants



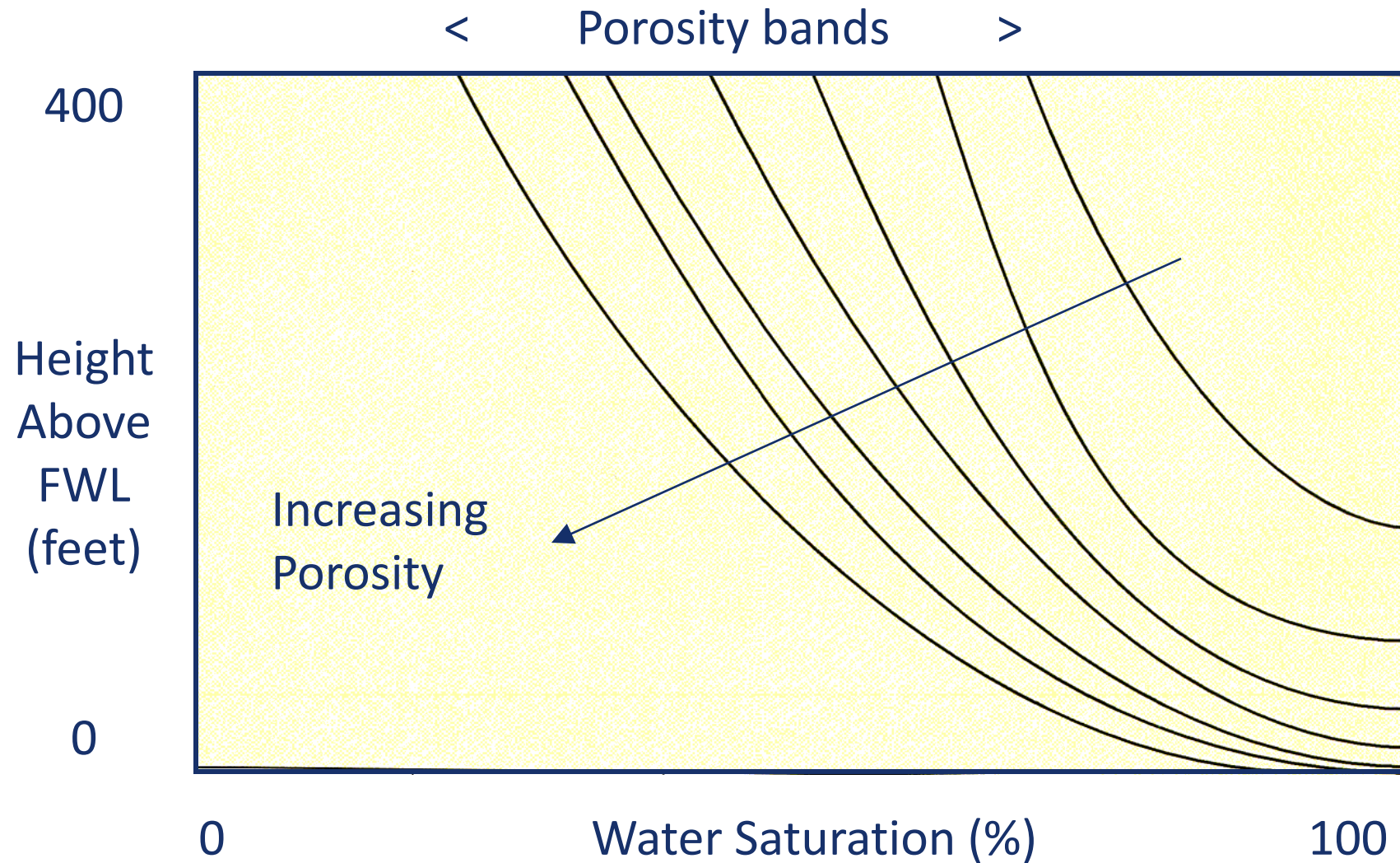
# Water Saturation vs. Height Data

What do we see in the well data?



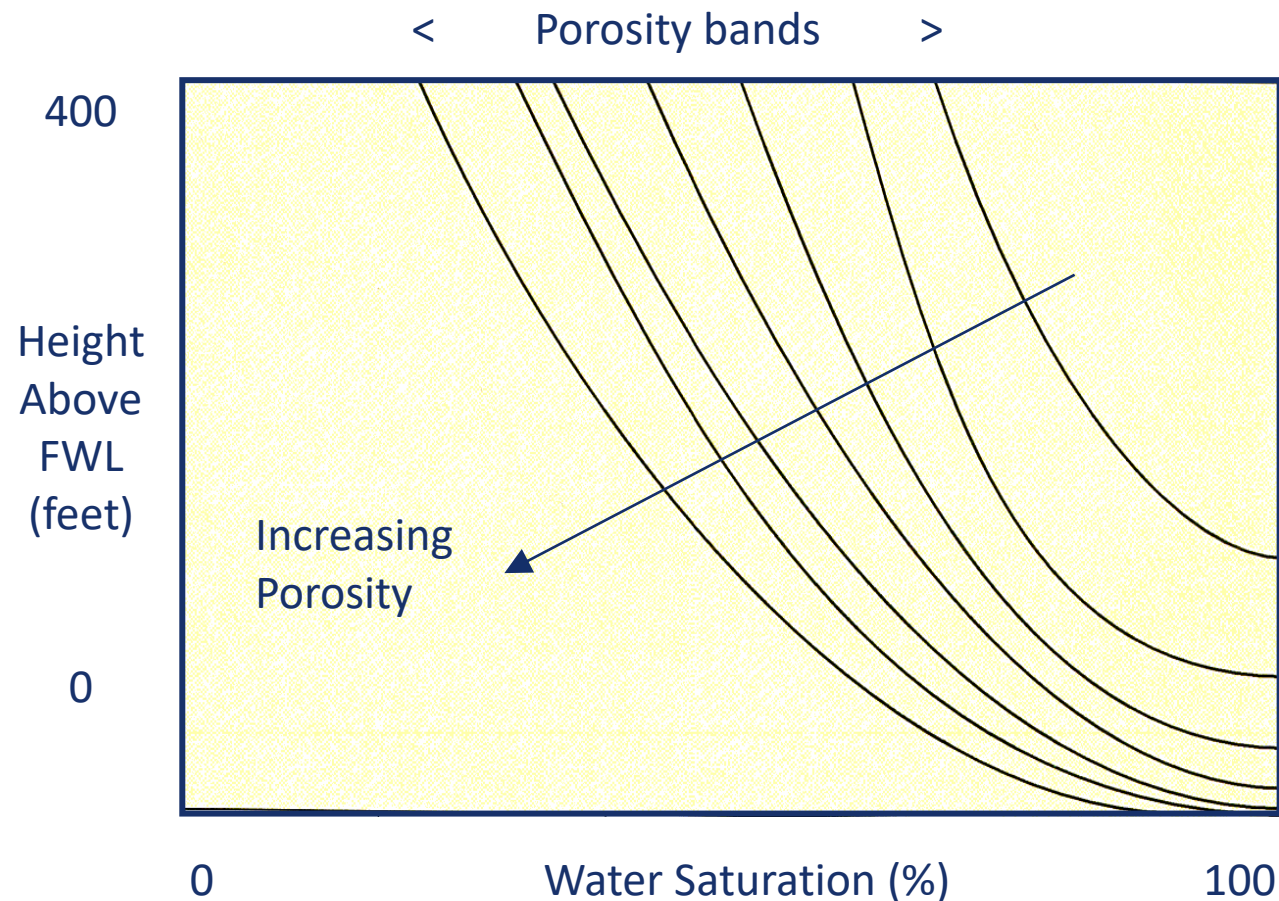
Source – Southern North Sea Gas field

# Classical Water Saturation vs. Height Curves



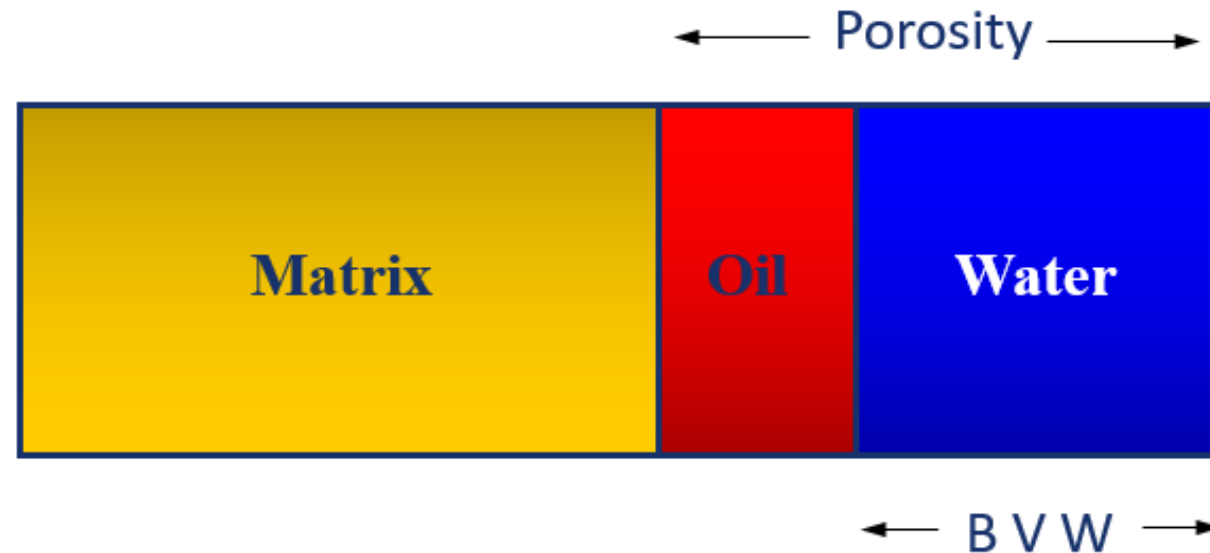
# Problems with Classical Swh Functions

- Sufficient data are required for each porosity band
- Defining the pore entry pressure (threshold height) can be difficult
- Visually and mathematically unconvincing



# The Bulk Volume of Water (BVW)

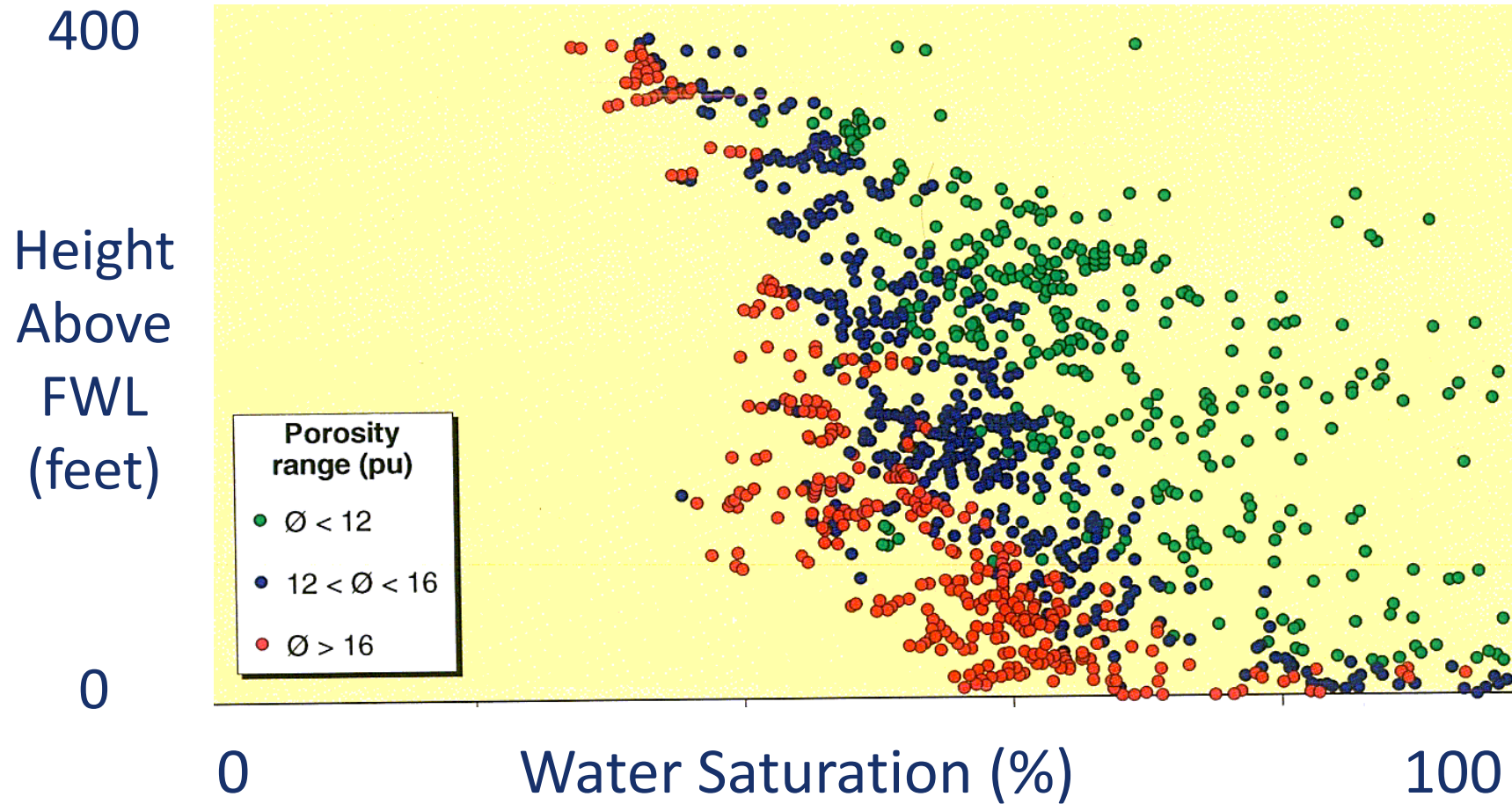
Bulk Volume of Water = Porosity x Water Saturation



B V W = % volume of water in a unit volume of reservoir

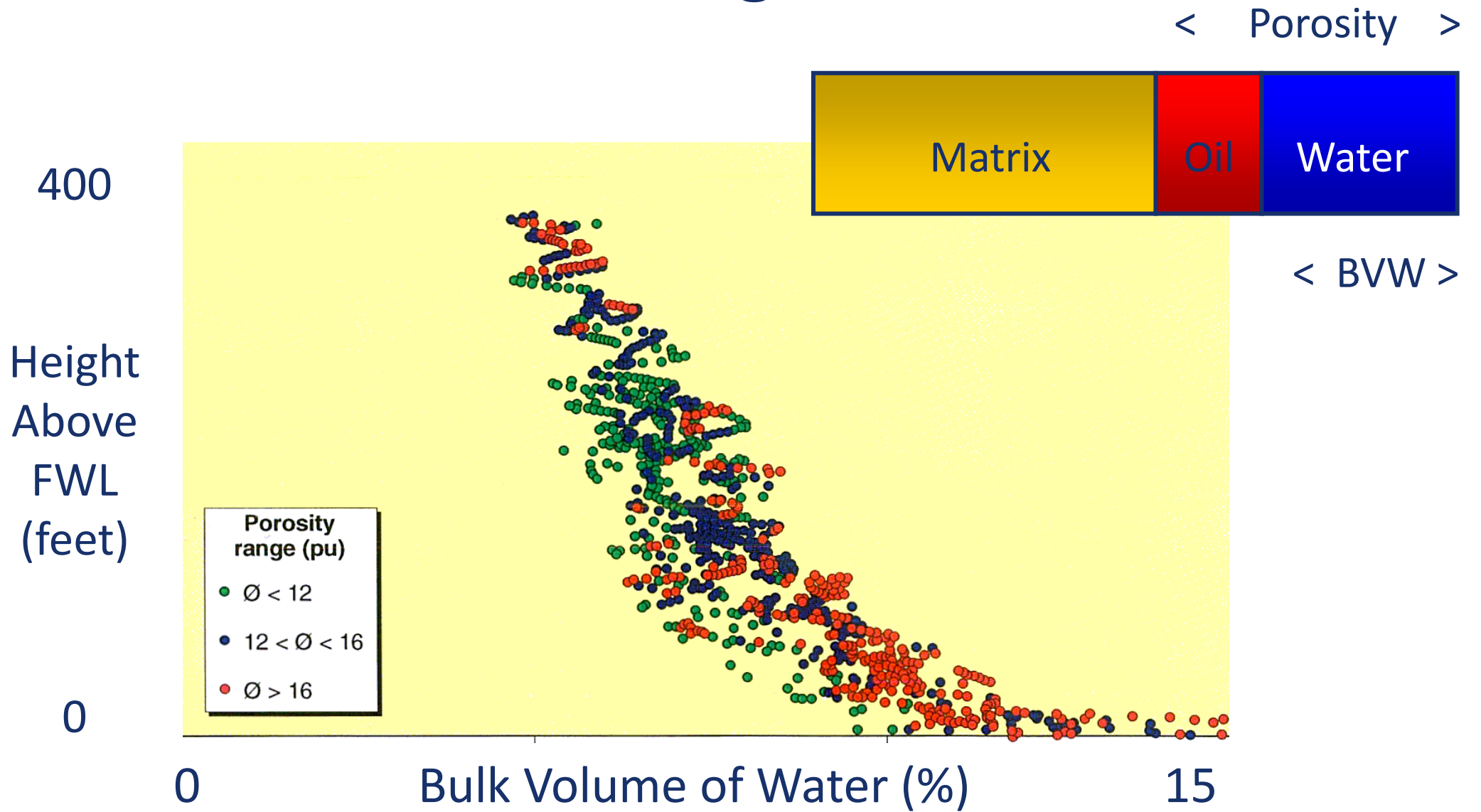
This is what is measured by electrical logs and by core analysis

# Water Saturation vs. Height Data



Source – Southern North Sea Gas field

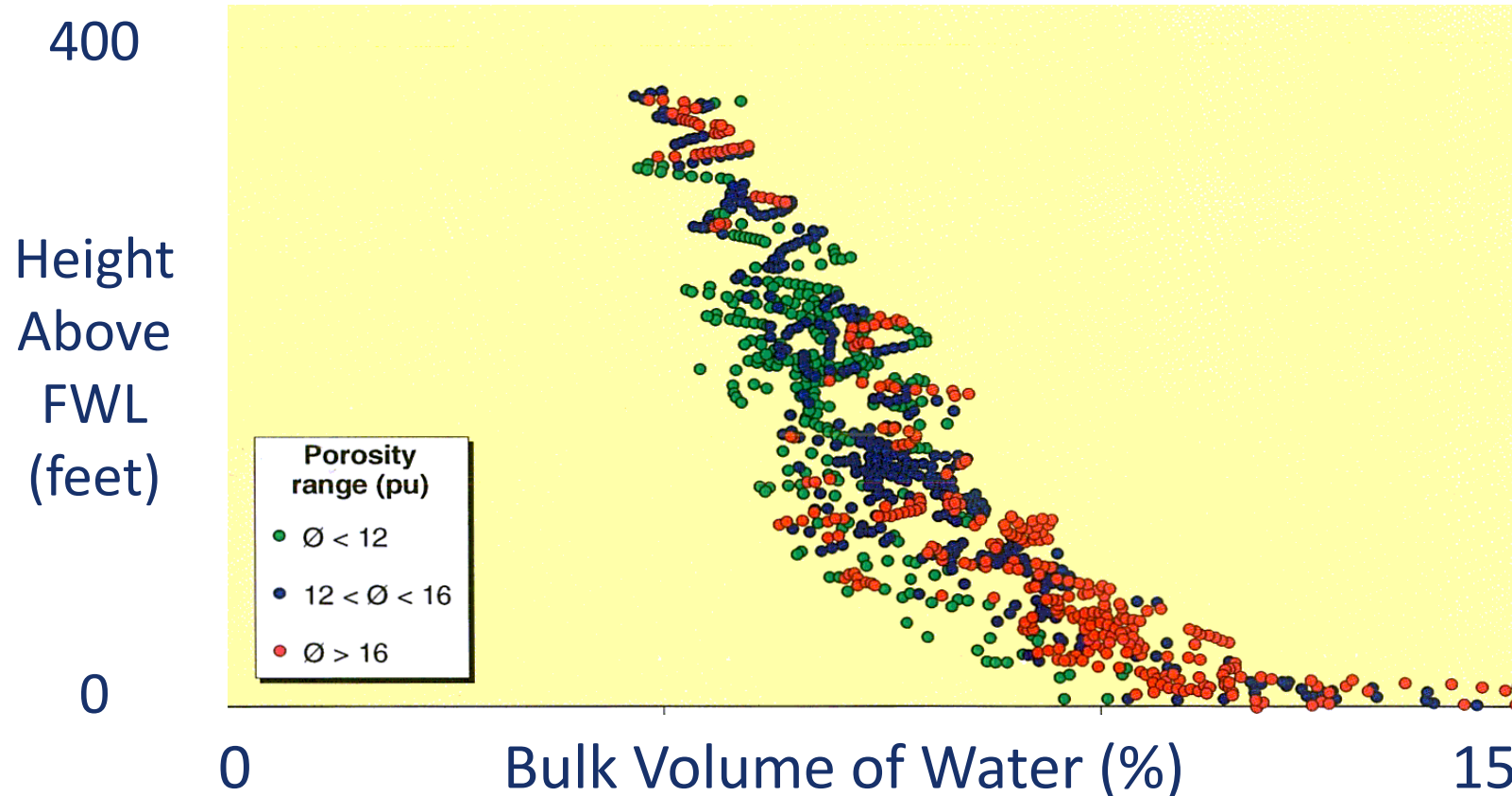
# Bulk Volume of Water vs. Height Data



# BVW is Independent of Rock Properties

The bulk volume of water is independent of rock properties

Can be verified by simply plotting facies-type, porosity or permeability on the z-axis (colour) on the cross-plot



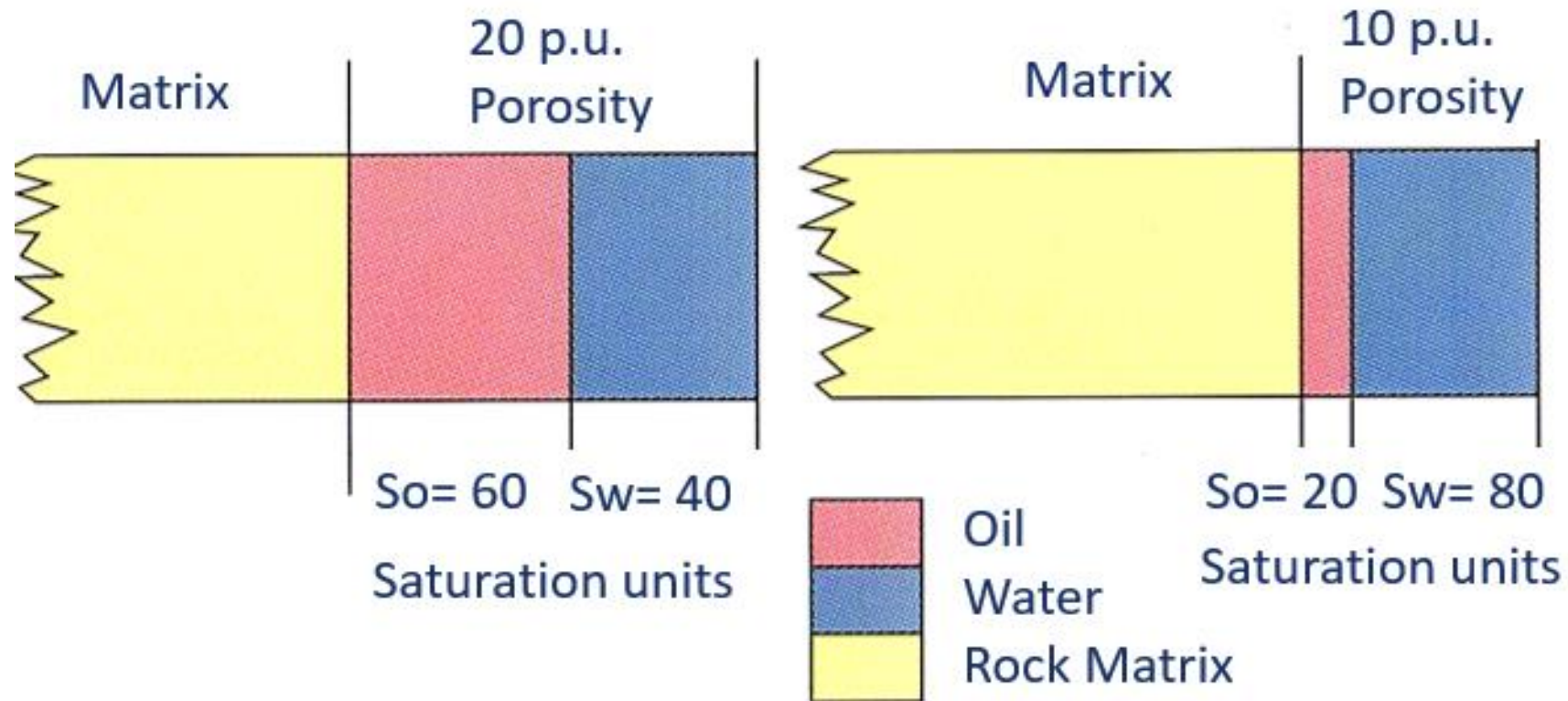
# Net Reservoir Cut-off

- Required for upscaling parameters for the reservoir model
- Net Reservoir
  - The portion of reservoir rock which is capable of **storing** hydrocarbon
  - Relatively easy to pick
  - Usually based on a porosity cutoff
- Net Pay
  - “The portion of reservoir rock which will **produce** commercial quantities of hydrocarbon”
  - Often used to select perforation intervals
  - Very difficult to pick
  - Depends on the **oil price**?



# What the fractal function tells us about net reservoir

Bulk Volume of Water = Function (Height above the FWL)



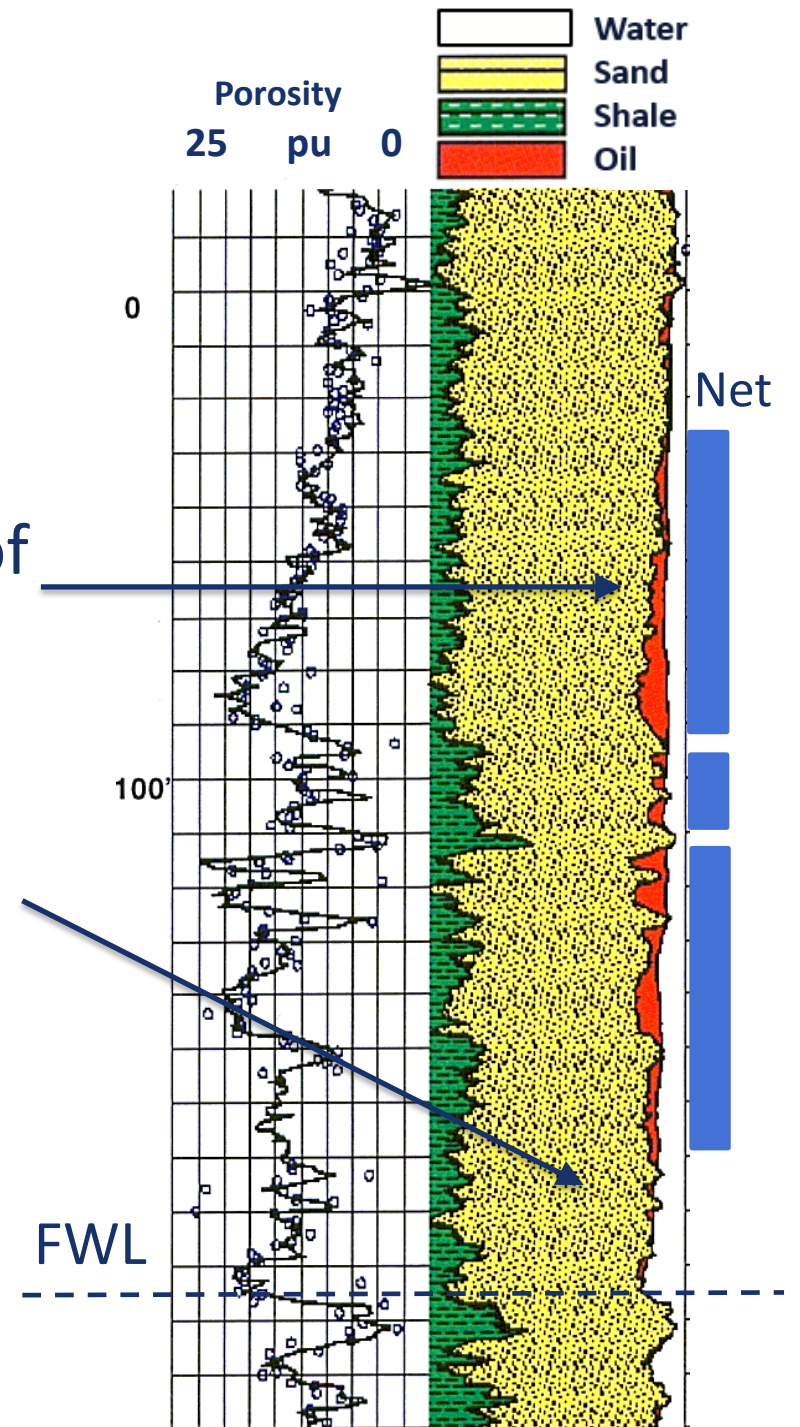
The BVW fractal function gives the net reservoir cutoff  
In this example: porosity > 9 porosity units

# Net Reservoir Example

The Net Reservoir cut-off **varies** as a function of height above the free water level (FWL)

Reservoir high above the FWL has low saturations of capillary bound water and hydrocarbon enters the smaller pores

Reservoir just above the FWL, with **higher** porosities, contains high saturations of capillary bound water and there is a no room available for hydrocarbons

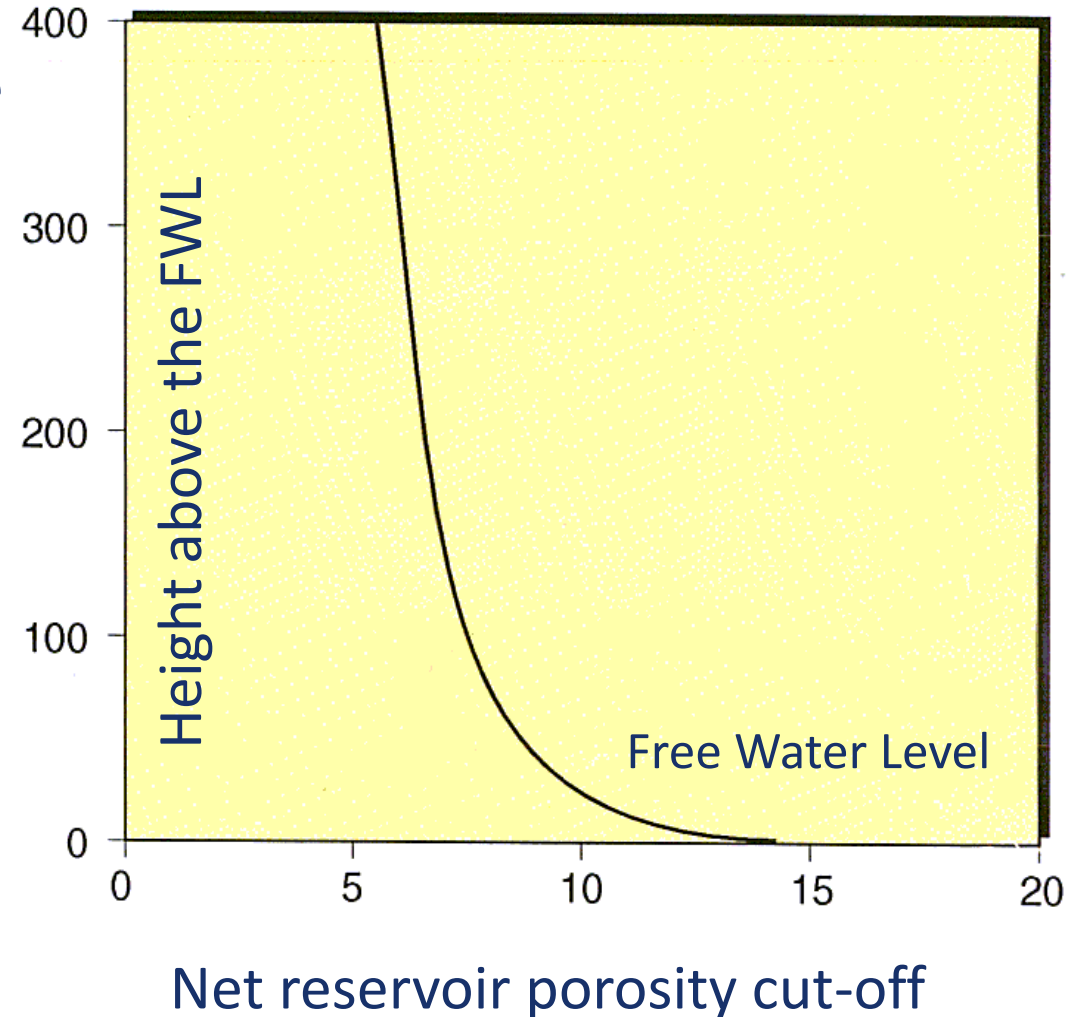


# Net Reservoir Cut-off

Net reservoir is defined as the rock capable of holding hydrocarbon

The net cut-off is required for averaging porosity and water saturation in the reservoir model

The net reservoir cut-off **varies** as a function of height above the FWL



# The Fractal Water Saturation vs. Height Function

$$BVW = aH^b$$

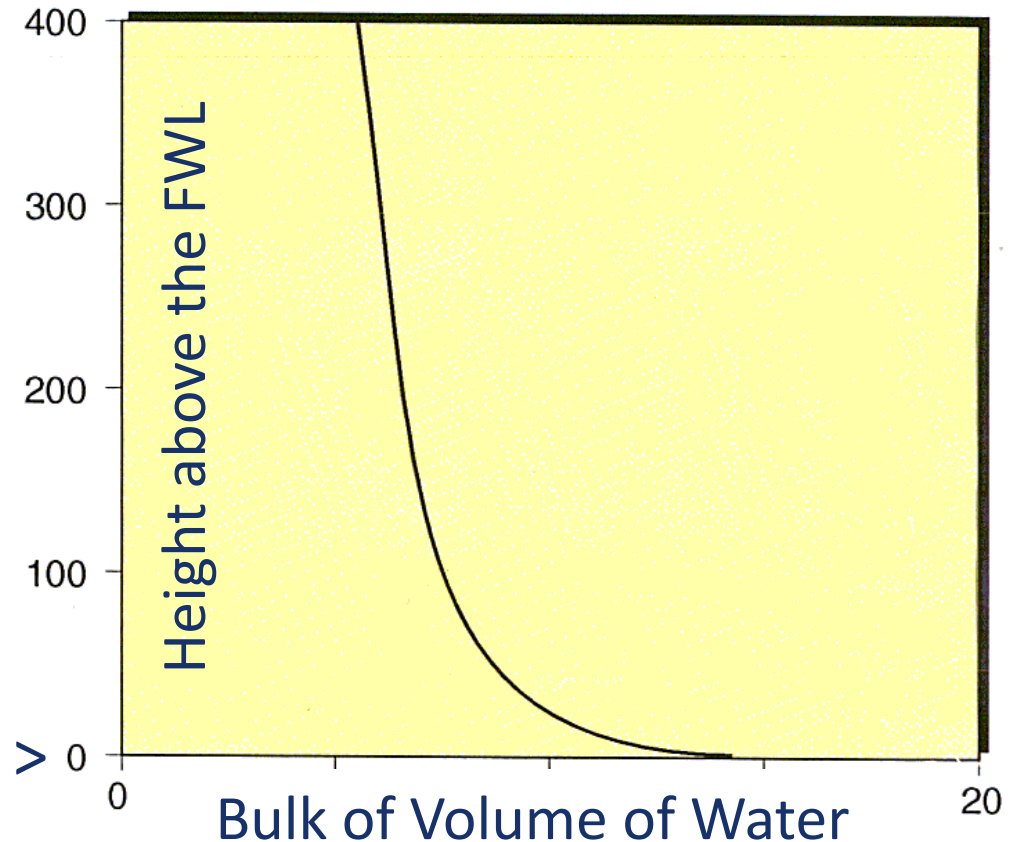
Where:

$BVW$  = Bulk Volume Water ( $S_w * \Phi$ )

$H$  = Height above FWL

$a, b$  = Constants

Free Water Level > 0



- Derived from the fractal nature of reservoir rocks
- Based on the bulk volume of water (BVW)
- Independent of facies type, porosity and permeability
- **Two** parameters completely describe your reservoir

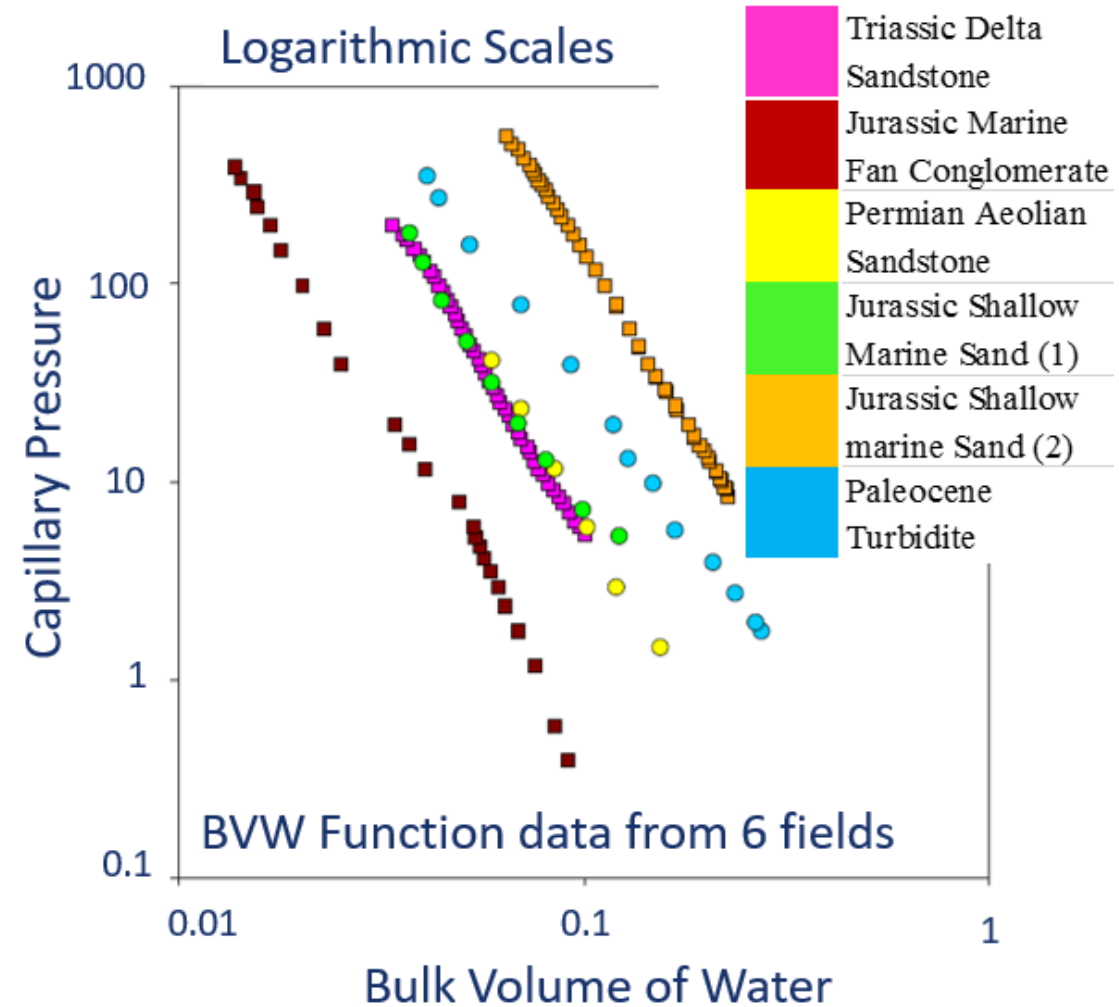
# The Fractal Function is easily calculated

$$BVW = a H^b$$

The BVW function is a straight line when plotted on **log** scales

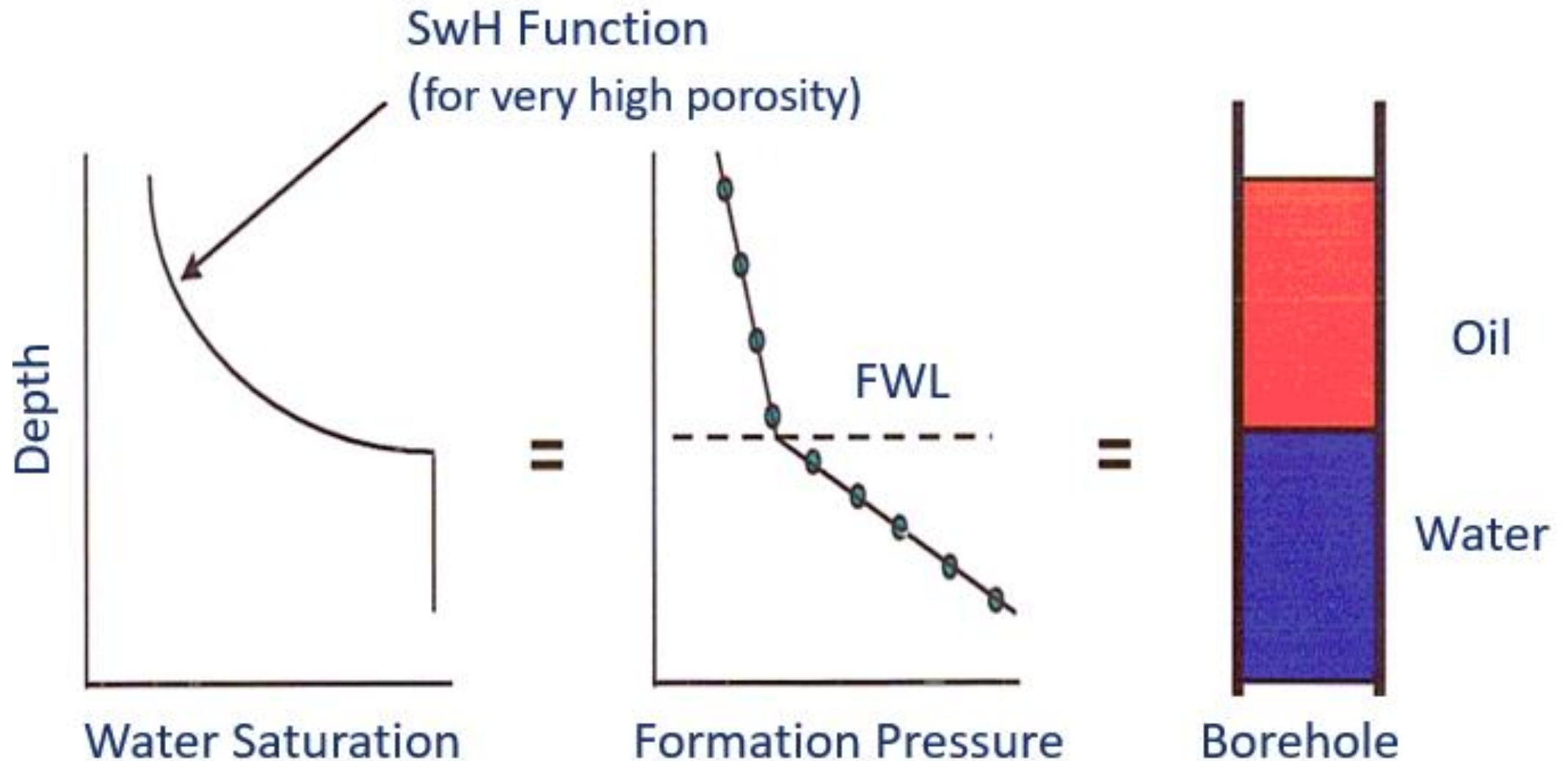
$$\log BVW = \log a + b \log H$$

Only 2 valid core or electrical log points required to calculate the constants 'a' & 'b'



# The Free Water Level (FWL)

FWL is the **horizontal** surface of zero capillary pressure



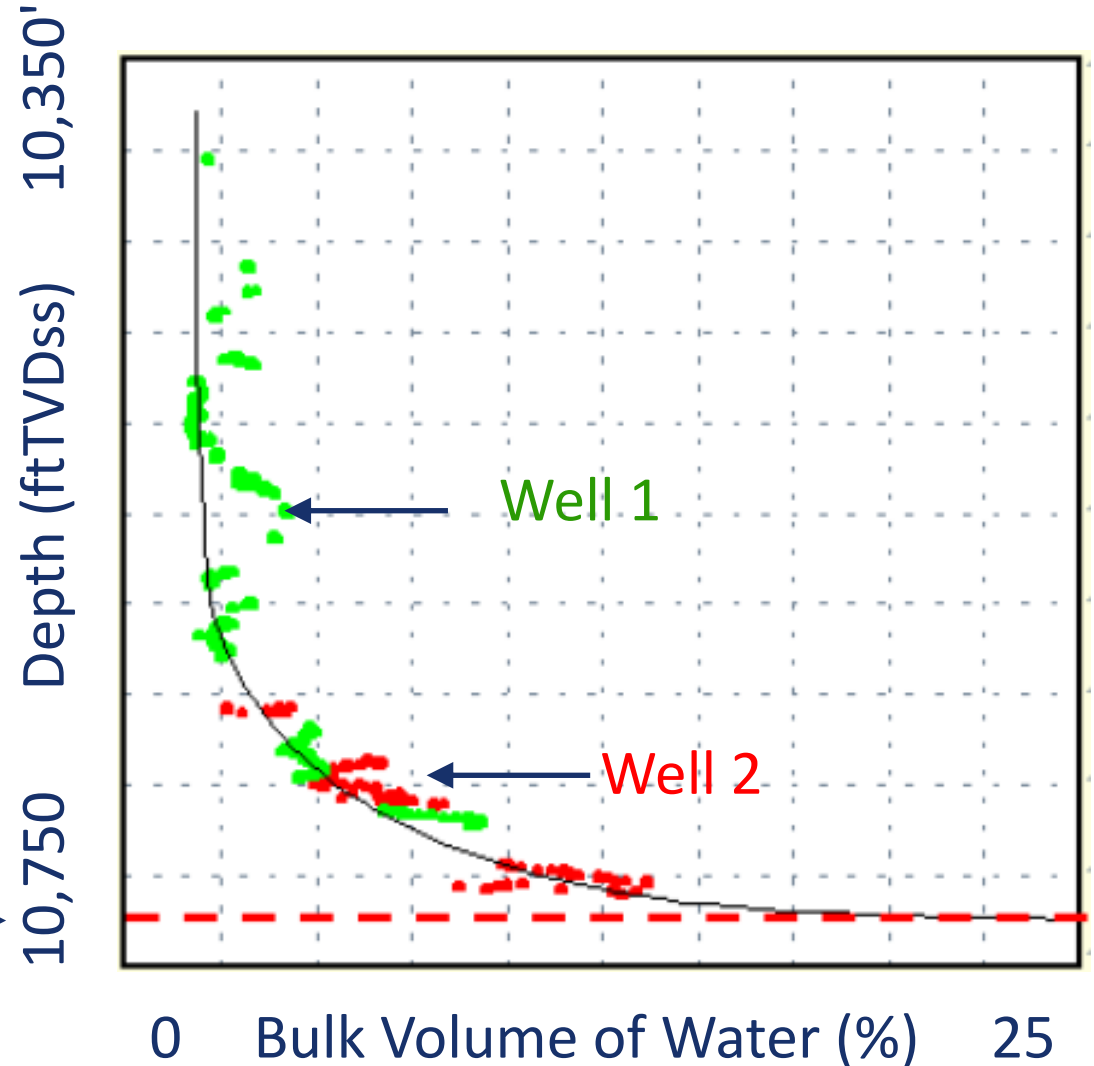
# Picking the Free Water Level

North Sea Oil Field

Two wells that **don't** intercept the FWL

BVW trend identifies the FWL and confirms the wells are probably in the same compartment

FWL=10,730 ftTVDss

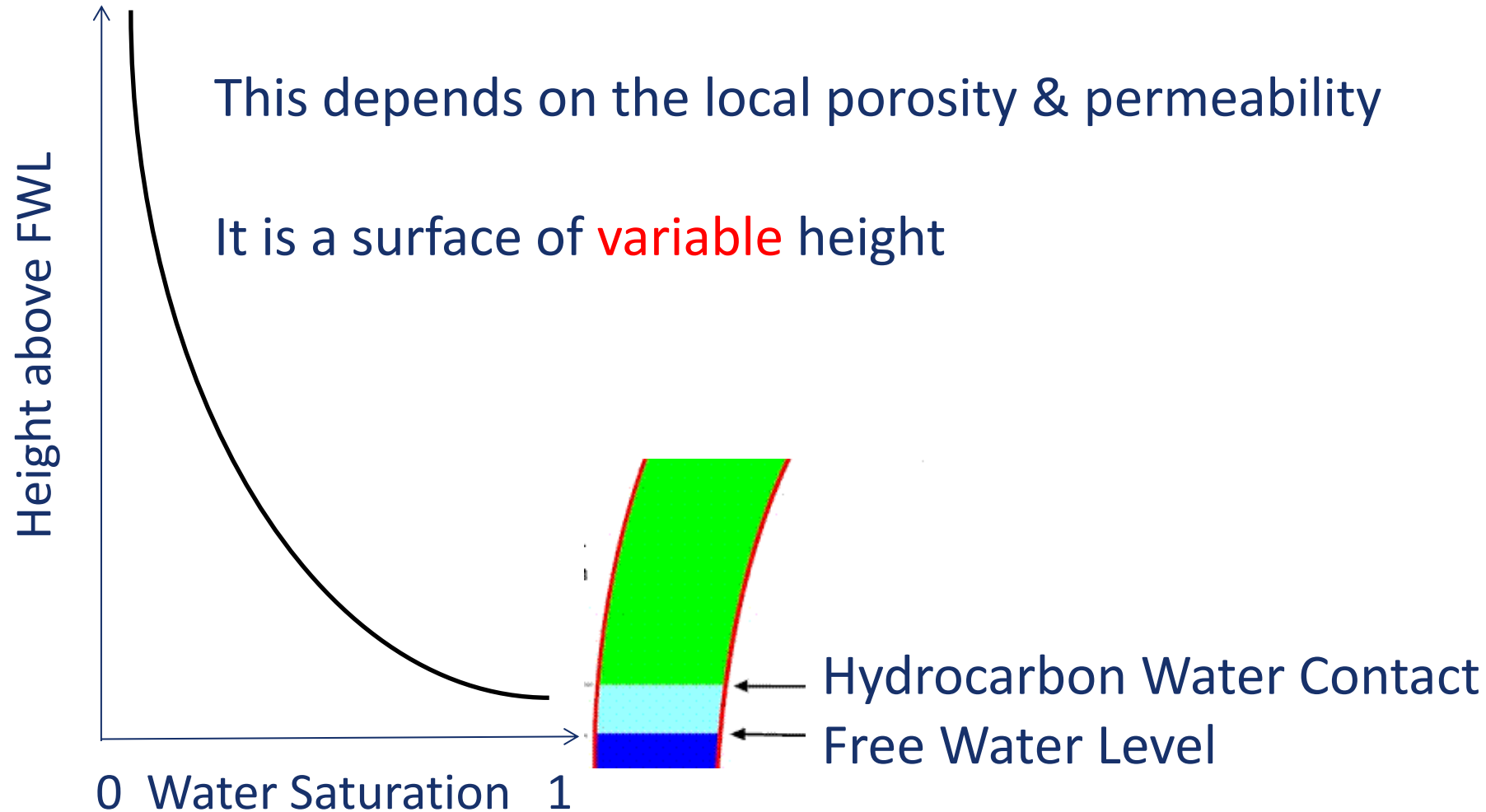


# Hydrocarbon Water Contact

The HWC is the height where the pore entry pressure is sufficient to allow hydrocarbon to start invading the formation pores

This depends on the local porosity & permeability

It is a surface of **variable** height





# Hydrocarbon to Water Contact Determination

$$S_w = \frac{\text{Bulk volume of water}}{\text{Porosity}}$$

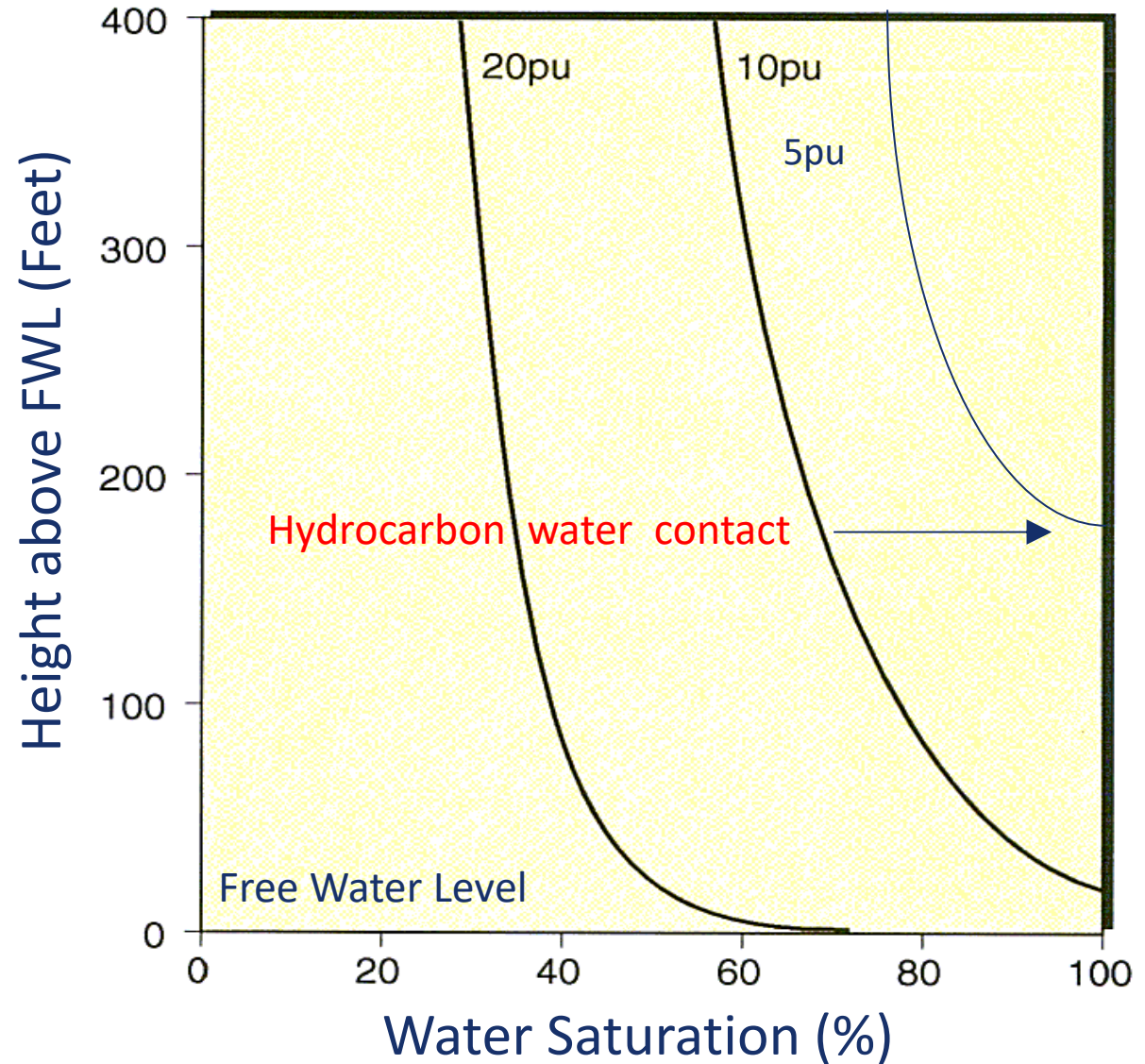
$$S_w = \frac{aH^b}{\text{Porosity}}$$

Where:

$H$  = Height above FWL

$a, b$  = Constants

The fractal  $S_w h$  function gives the hydrocarbon water contact as a function of porosity



# Sw vs. Height modelling

Required to initialise the reservoir model

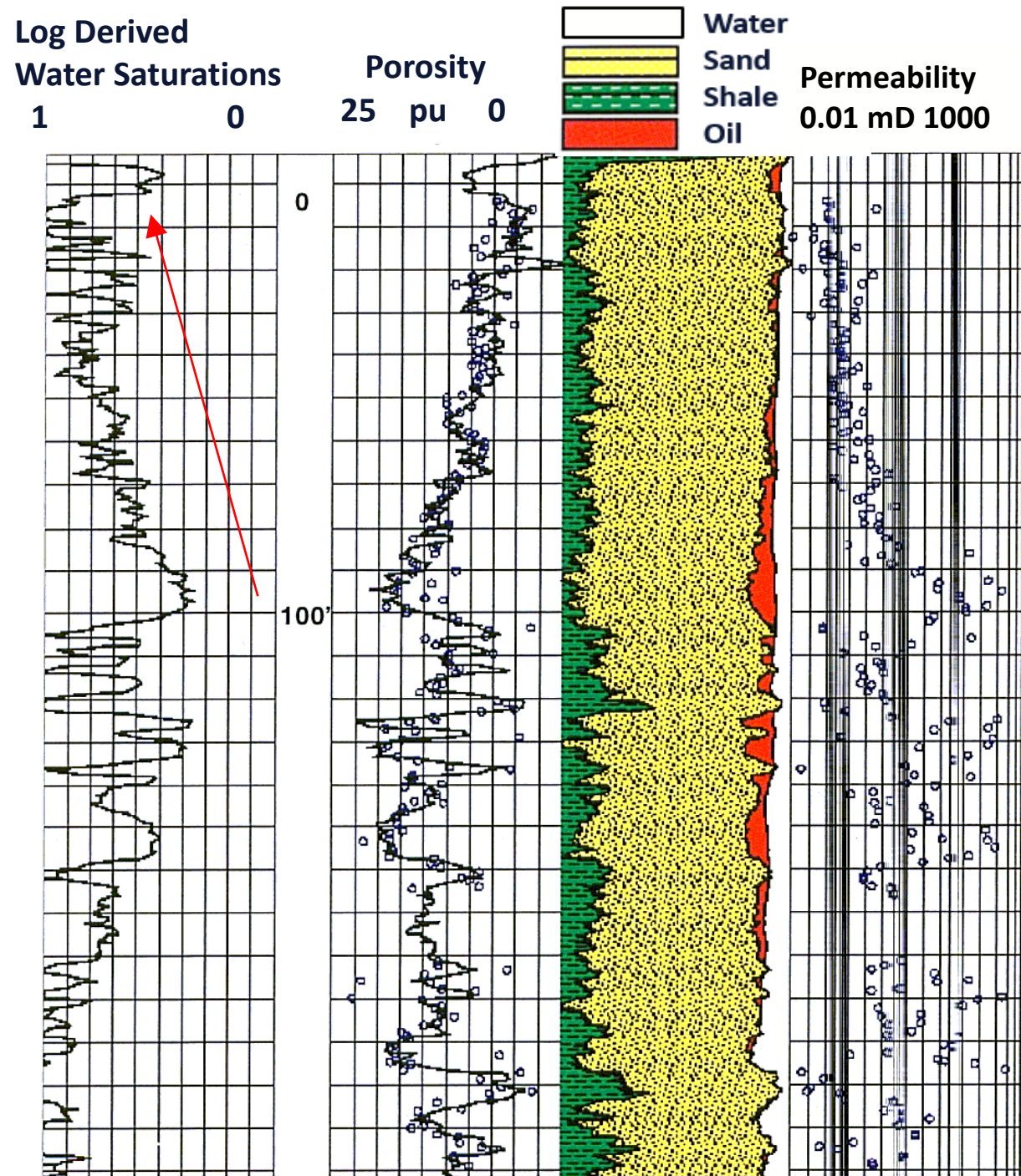
North Sea Case Study

London Petrophysical Society  
supplied data

Very difficult data set

Heterogeneous formation

Sw **increases** with height!



# Sw vs. Height modelling Results

$$BVW = aH^b$$

Where:

$BVW$  = Bulk Volume Water ( $S_w * \Phi$ )

$H$  = Height above FWL

$a, b$  = Constants

Good match in all litho-facies types

Permeability not required

Defined by only 2 parameters

Do we always need resistivity logs?

Fractal & Log Derived

Water Saturations

1

0

Porosity

25

0



Water

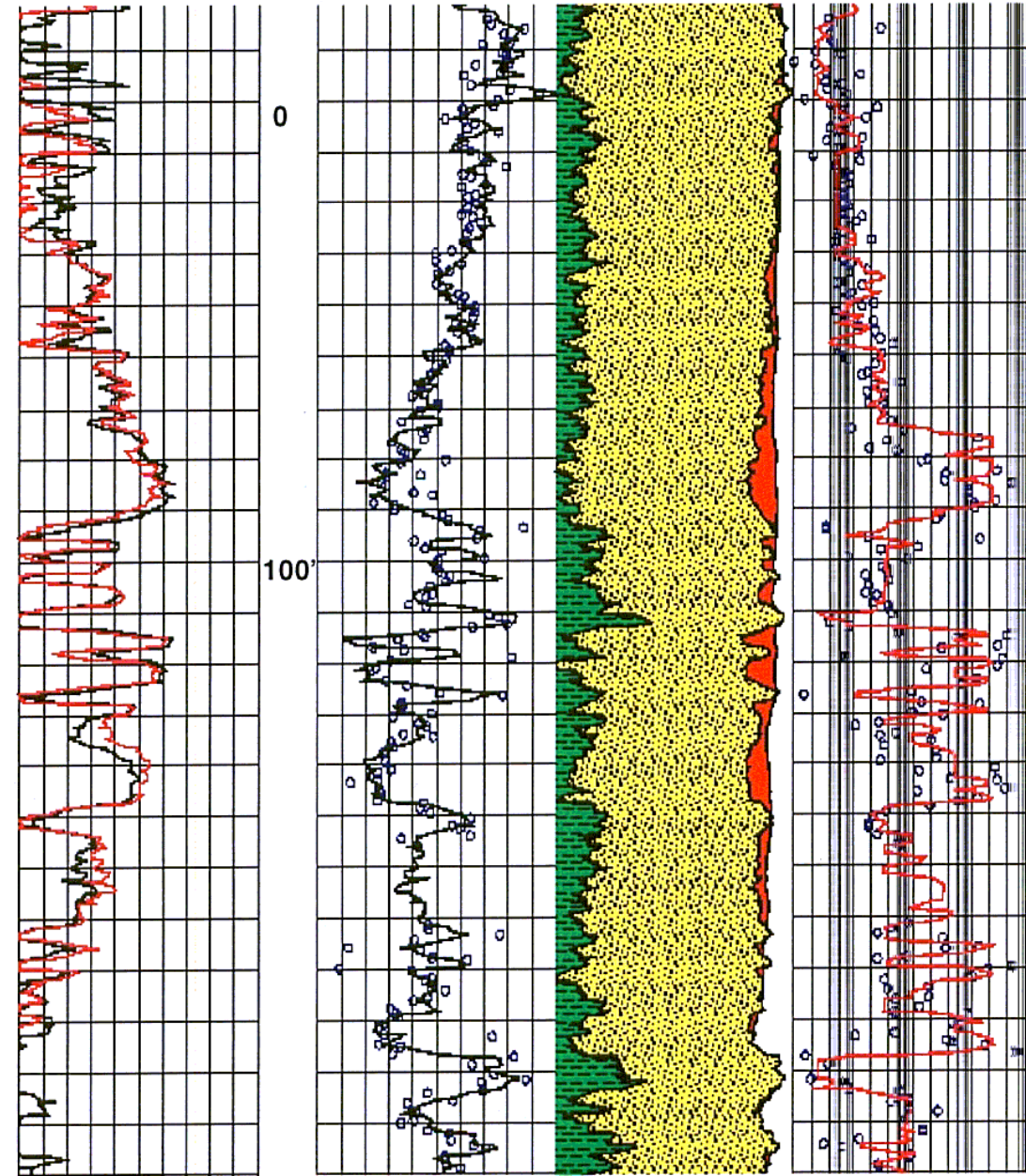
Sand

Shale

Oil

Permeability

0.01 mD 1000

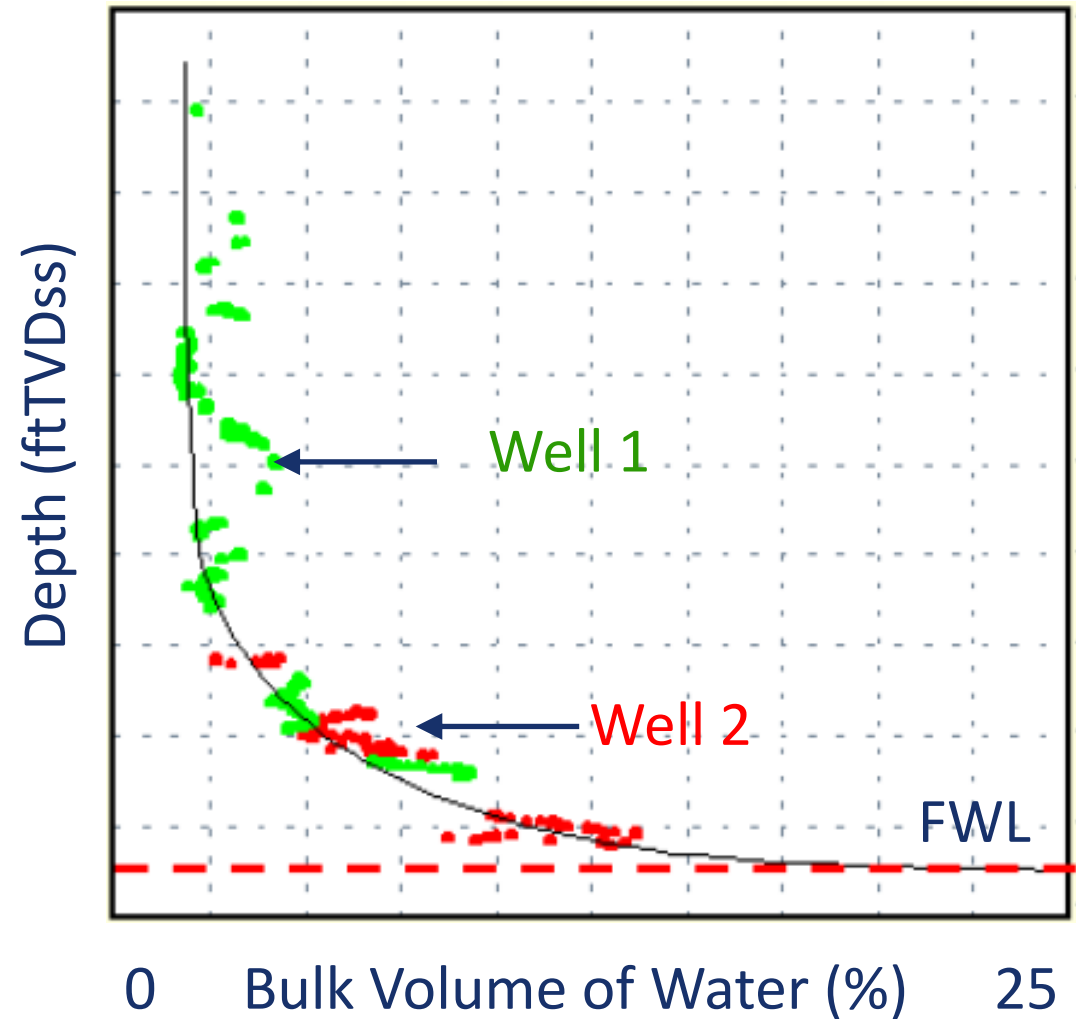


# Using the Fractal Function for Depth control

Depth is the most important downhole measurement

True vertical depth subsea can be +/- 30 feet

Identification of the FWL normalises well depths



# The Differential Reservoir Model

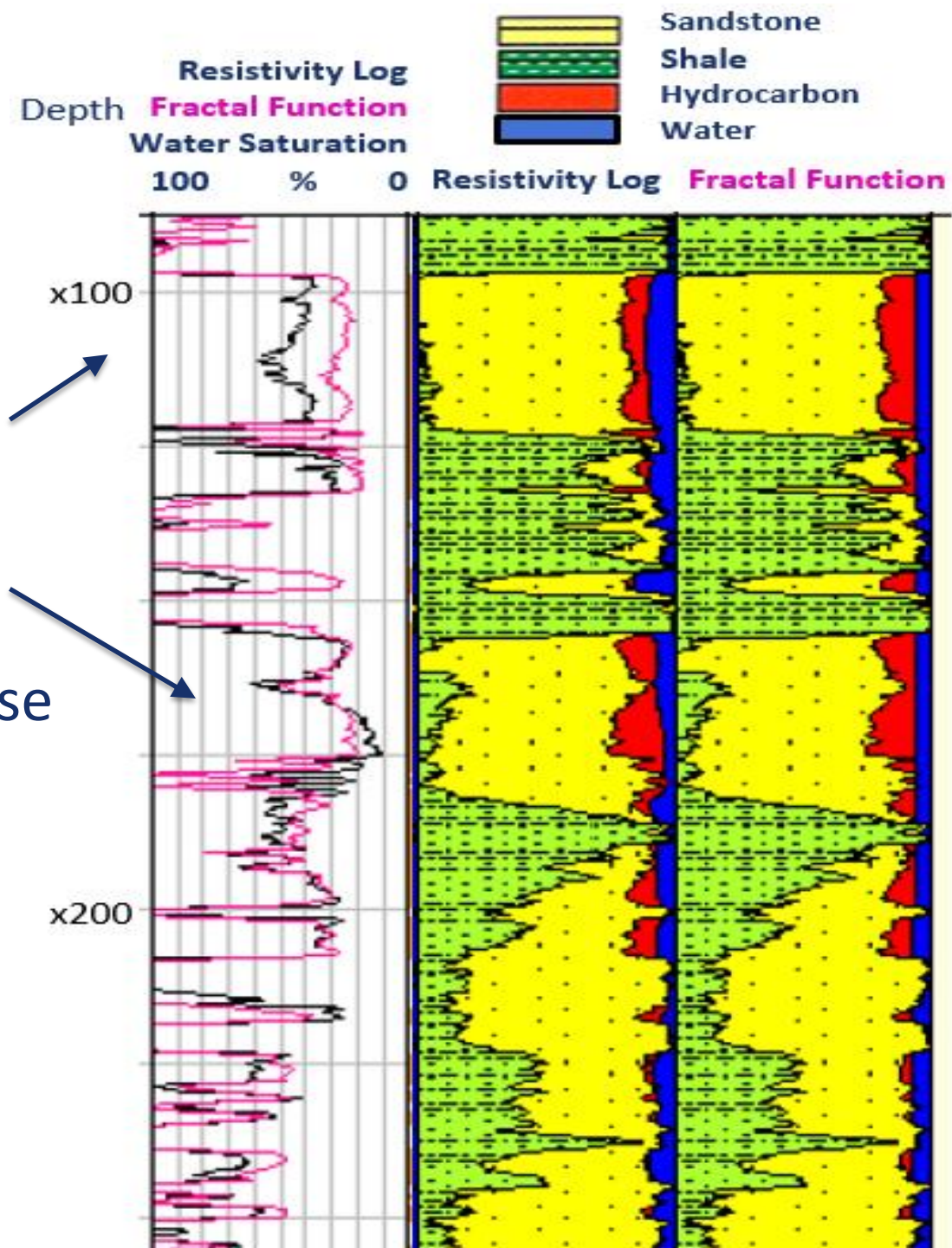
Comparison between resistivity and fractal derived water saturations

Swept zone showing residual oil saturations

By-passed hydrocarbon

The resistivity log is **incorrect** in thin beds, close to bed boundaries and where there are conductive shales

The fractal function ignores thin beds, bed boundaries and shales



# Irreducible Water Saturation (Swirr)

Is the lowest water saturation that can be achieved in a core plug

This is achieved by flowing hydrocarbon through a sample or spinning the sample in a centrifuge

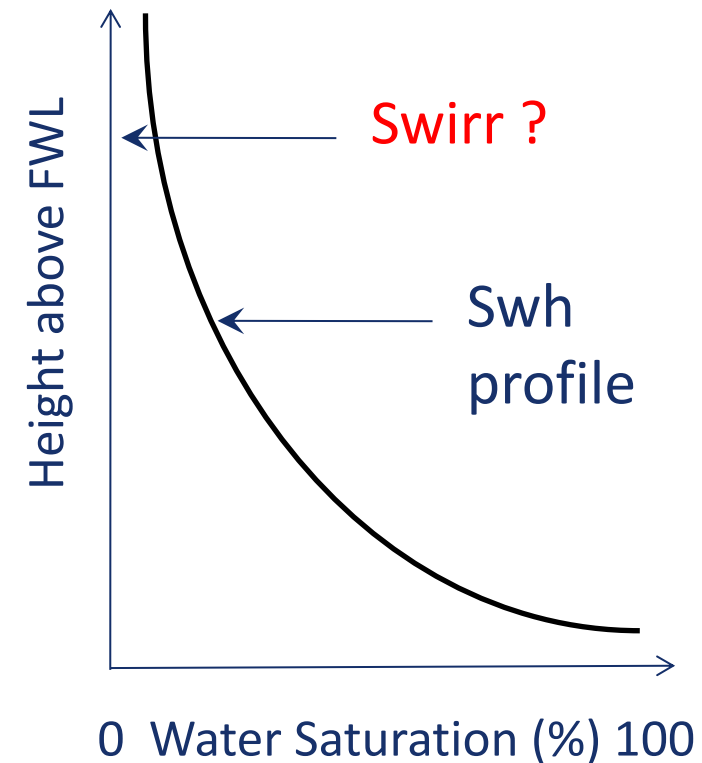
This depends on the drive pressure or the centrifuge speed

Water saturation therefore depends on the height above the free water level

A minimum Swirr does **not** exist

The transition zone extends indefinitely

The Fractal Swh function determines Swirr as a function of height and porosity

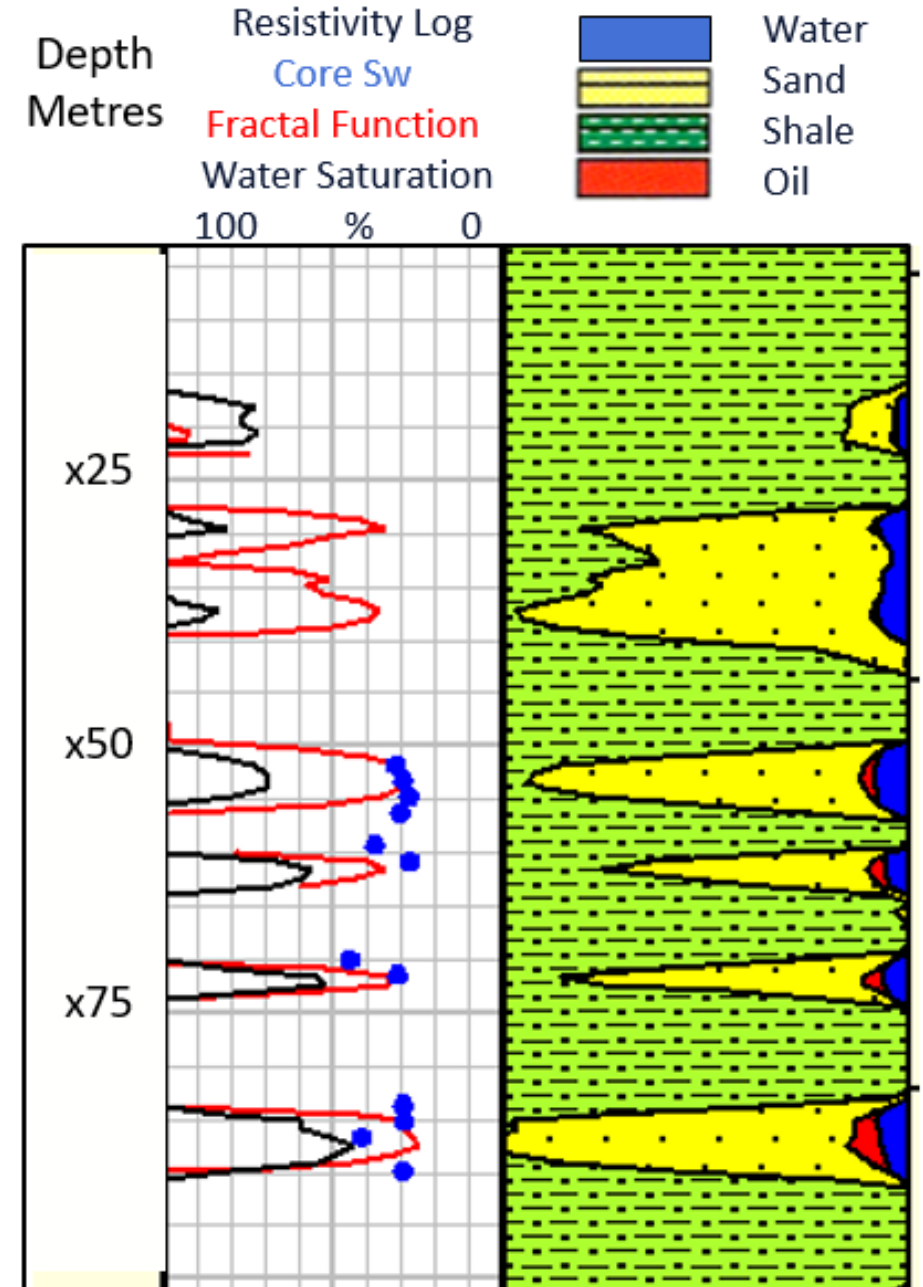


# Core water saturations

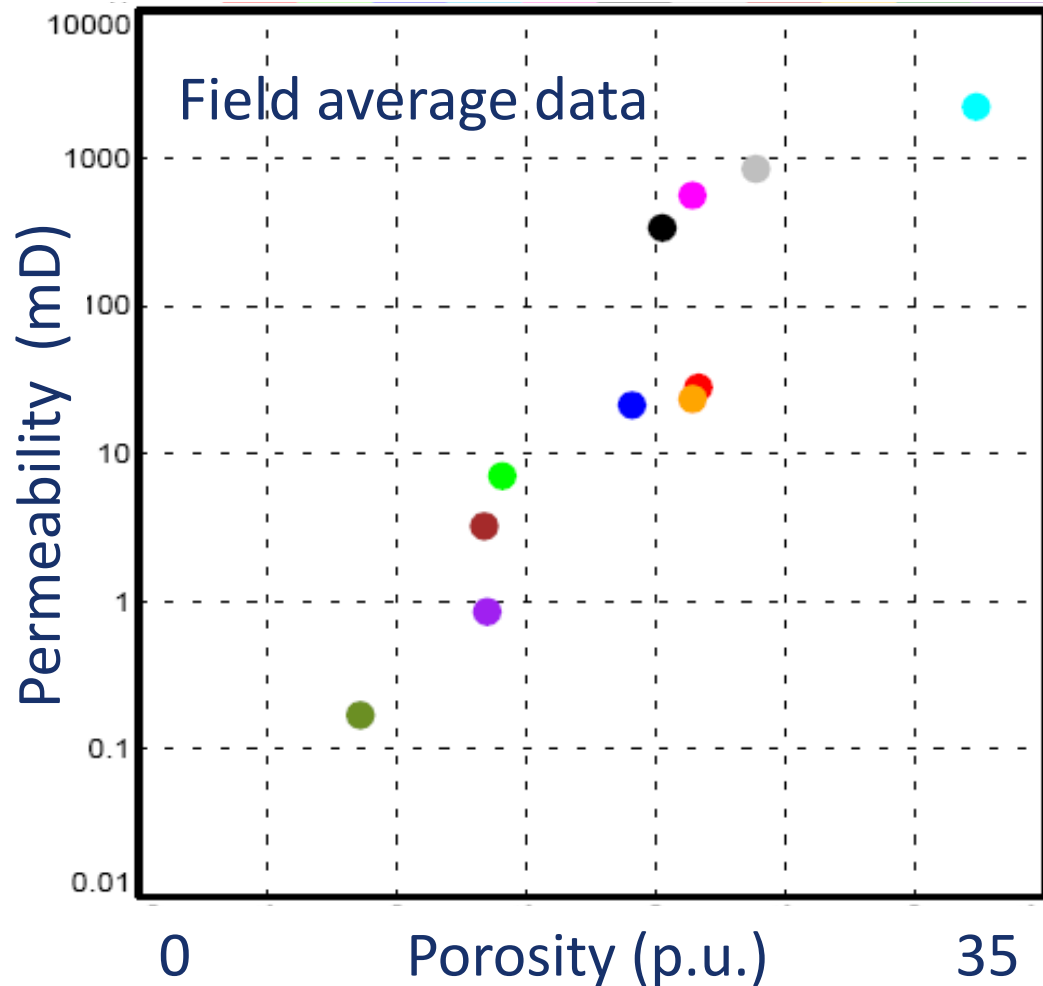
Accurate core water saturations

- Well drilled with oil base mud doped to identify any mud filtrate contamination
- Cored above the FWL where the capillary bound water is immobile
- Only cores centres sampled

The core **confirms** the water saturations determined by fractal function



# Log and core data from 11 North Sea fields compared



Field	Fluid	Type	Porosity (pu)	Perm (mD)
	Gas	Permian Fluvial	9	0.2
	Oil	M. Jurassic Deltaic	13	3
	Oil	Devonian Lacustrine	14	7
	Gas	Permian Aeolian	14	0.9
	Oil	Palaeocene Turbidite	20	21
	Gas	Permian Aeolian	20	341
	Gas Condensate	L. Cretaceous Turbidite	24	847
	Oil	U. Jurassic Turbidite	21	570
	Oil	Palaeocene Turbidite	21	24
	Oil	Palaeocene Turbidite	22	27
	Gas	Palaeocene Turbidite	32	2207



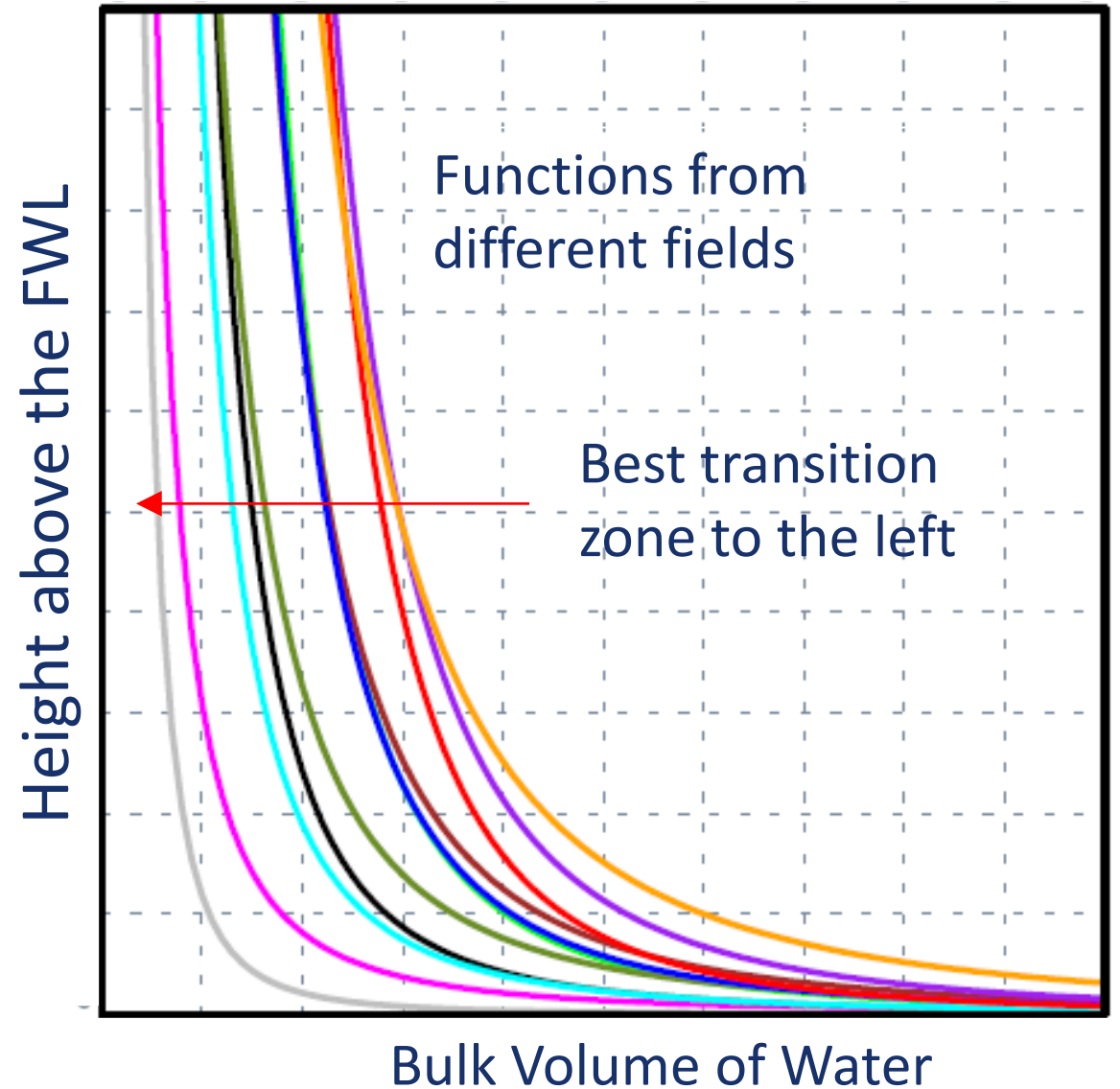
# Case Study Observations

Transition zones compared

Which is the best  $S_{wh}$  function?

The shape of the transition zone is related to pore geometry rather than porosity or permeability alone

Fractal Functions quantify the pore geometry

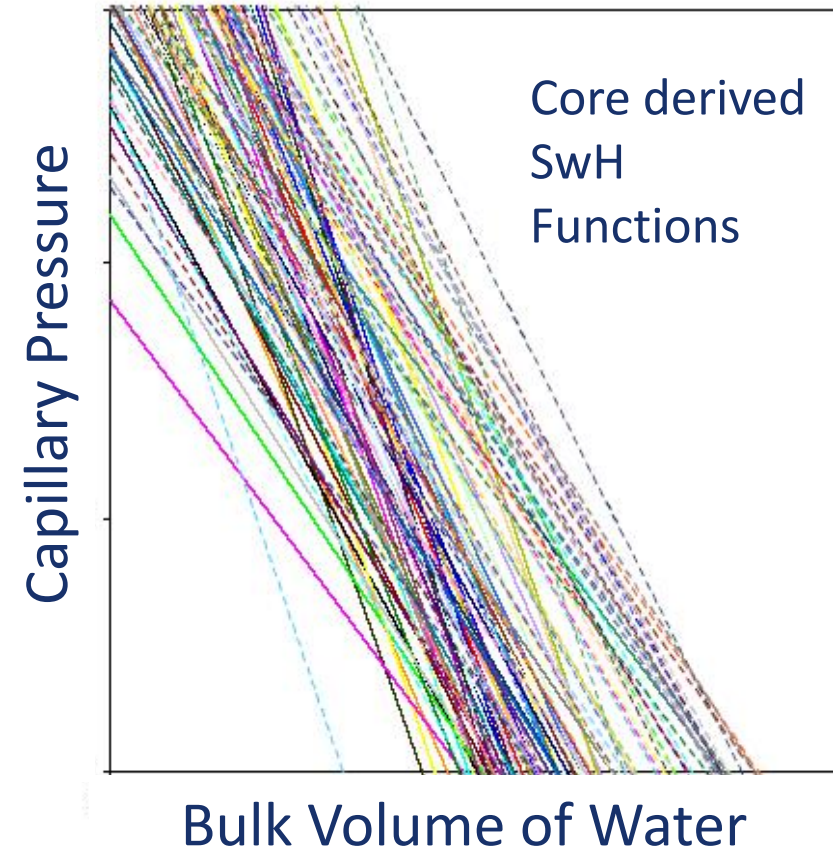
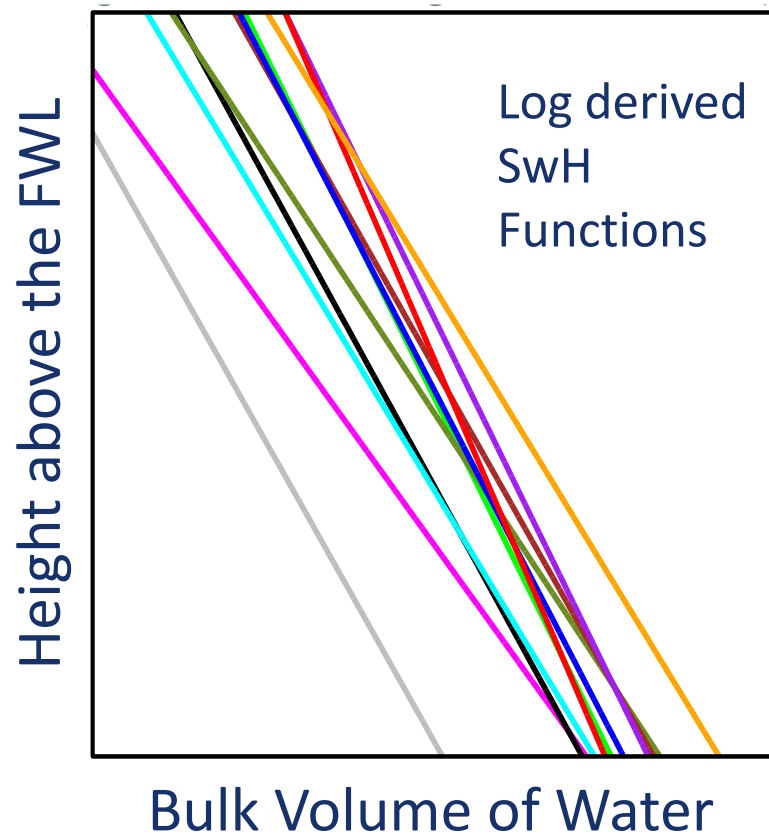


# Comparison between Log and Core BVW Functions

The Fractal Water Saturation vs. Height Function is linear on log-log scales

Electrical log and core functions are the **same** irrespective to whether they were determined from logs or core data

This confirms the **fractal** distribution of reservoir capillaries



# Conclusions – fractals and hydrocarbon volumes

- Sw<sub>h</sub> function derived from the fractal nature of reservoir rocks
- Can be derived from electrical log or core data
  - Using simple linear regression of a log-log plot
  - Logs and core give the same function. They QC each other
  - This confirms fractal distribution of reservoir capillaries
- Determines Free Water Level and Hydrocarbon Water Contact
- Defines the Net Reservoir Cut-off and Transition Zone
- Independent of rock characteristics
  - Facies type, porosity and permeability
  - You can forget about bed boundary effects and shaliness
- Simple implementation in your reservoir model

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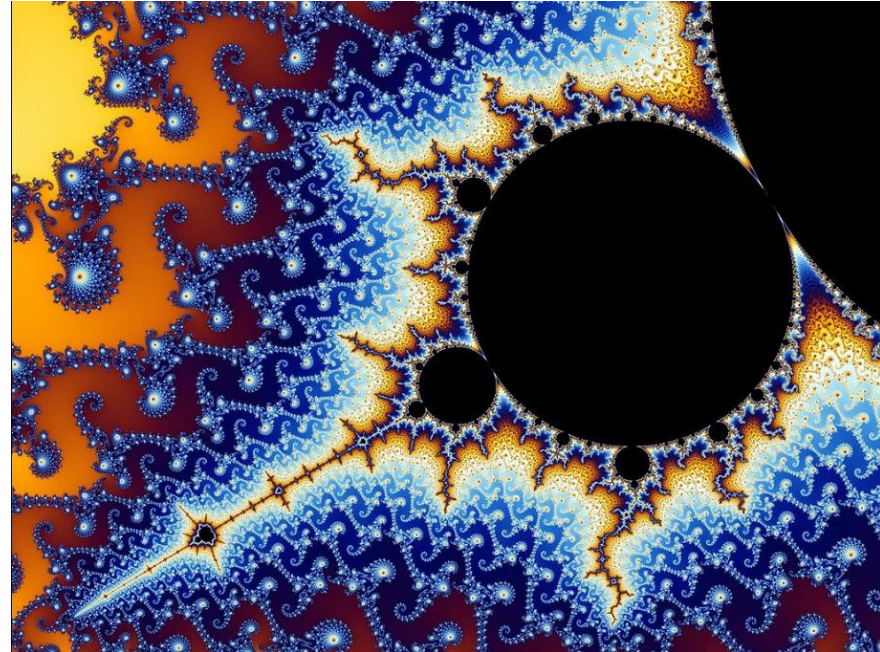


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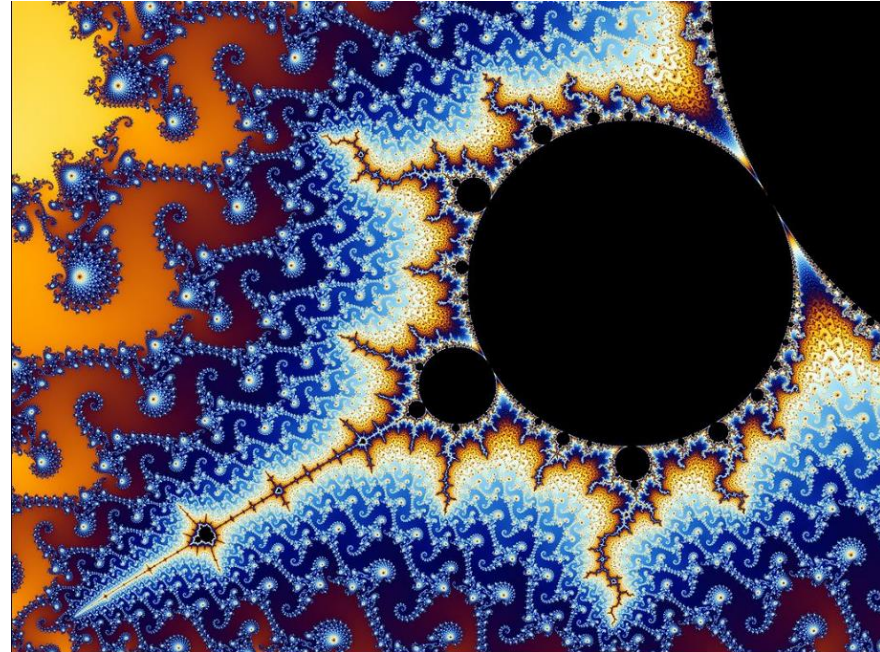
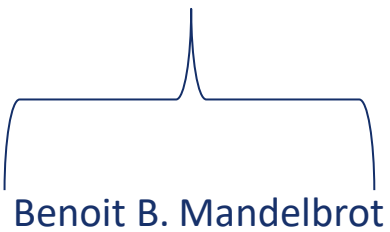
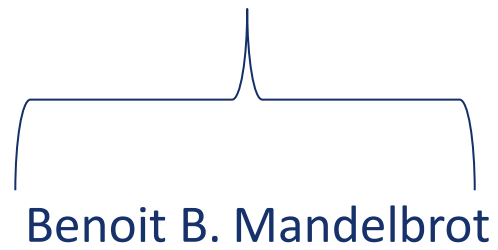
What's Benoit B. Mandelbrot middle name?

Benoit B. Mandelbrot



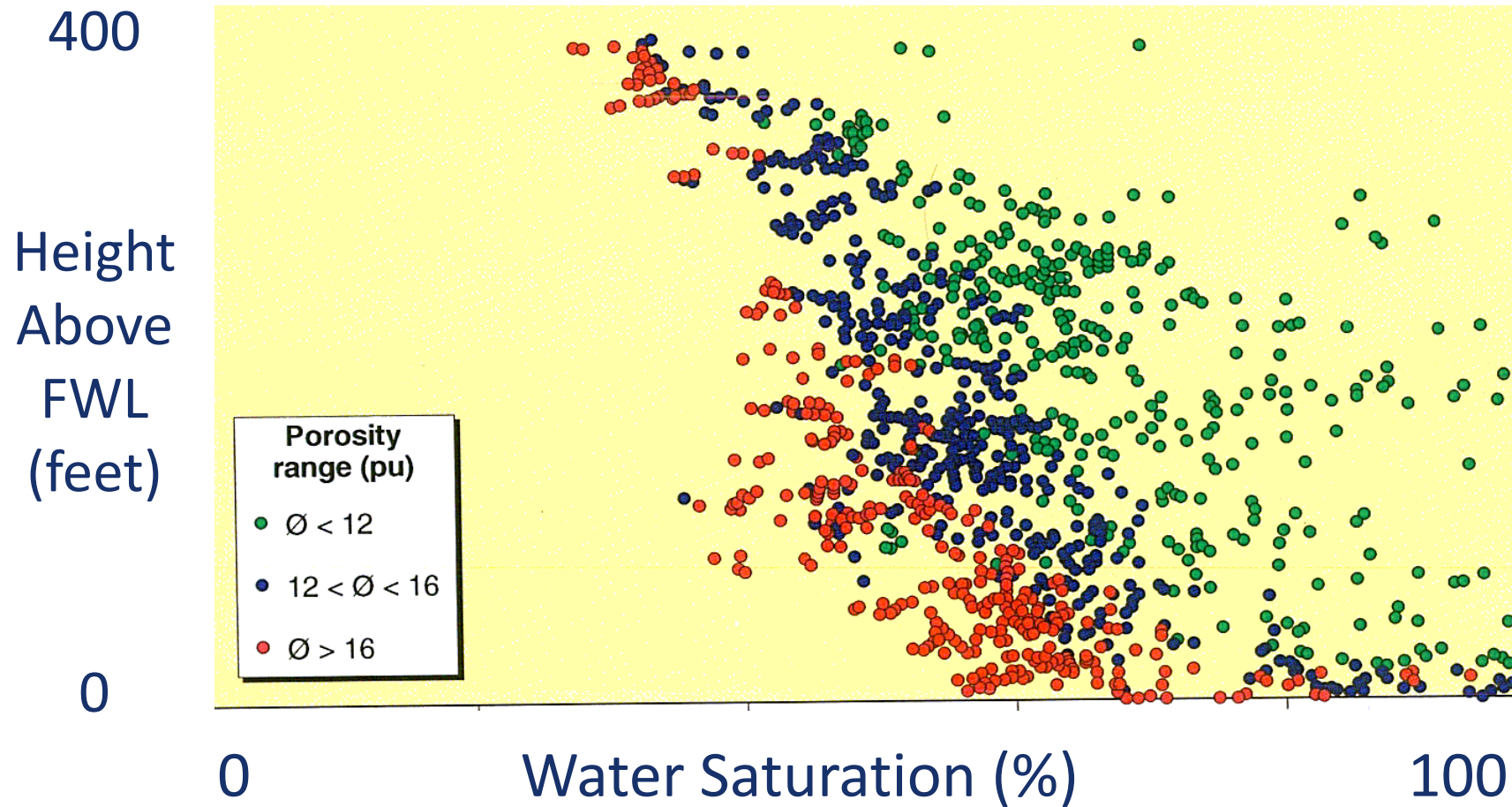
# What's Benoit B. Mandelbrot middle name?

Benoit B. Mandelbrot



# Water Saturation vs. Height Data

What do we see in the well data?



Source – Southern North Sea Gas field

# Bulk Volume of Water vs. Height Data

